Liquidity and Financial Intermediation[1]

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Abstract

In this paper, we examine how liquidity premia are affected by the supplies of various types of assets. Specifically, we examine how the supply of government debt affects the real interest rate and the liquidity premium in the economy. More importantly, we examine how the financial sector satisfies the demand for liquidity by transforming illiquid assets into liquid assets, and thereby how the private supply of liquid assets is affected by the supply of government debt. We also show that a rise in uncertainty (i.e., risk premia) hampers the ability of financial institutions to provide liquid assets, and hence may lead to a contraction in the supply of liquid assets in addition to a rise in the liquidity premium. Hence, this paper establishes a close connection between risk premia and liquidity premia. This feature arises naturally out of our model, and as well seems to be a reasonable characterization of current events. The end result is a general equilibrium model that endogenizes the supply of liquid assets and their rate of return. Using quarterly U.S. data from 1950 to 2009, we show that central predictions of the model find strong support in the data.

1 Introduction

We develop a general equilibrium model of the credit market which endogenizes both the quantity and price of short term public and private debt. Our model follows the approach taken in Bansal and Coleman (1996), which builds on the work of Gurley and Shaw (1955), Friedman (1969), Lucas and Stokey (1987), and Townsend (1987) . Bansal and Coleman (1996) present a model which highlights the special role of some assets in providing liquidity. They examine the implications of demand for liquidity in explaining the equity premium and the term premium in government bonds. Their model captures the idea that banks face penalties for failing to fulfill their liabilities to depositors. To avoid these penalties, banks back their deposits with nominal Treasury debt; Treasury debt therefore provides a non-pecuniary liquidity service (liquidity yield) which raises their market price and lower their real yield. In this paper, we allow commercial banks to hold both public and private debt. This allows private debt issuers, in addition to the Treasury, to borrow at a lower pecuniary rate as private debt also provides liquidity services. In this model, financial intermediaries issue short term debt to capture this liquidity yield; they borrow by issuing liquidity-providing short term debt and invest in non-liquidity providing risky long-term securities, and incur bankruptcy costs in case of default. The trade-off between earning the liquidity yield and the prospect of bankruptcy costs determines the supply of private short term debt and its pecuniary yield. The Treasury also earns the liquidity yield; however, it faces the prospect that it may have to raise distortionary taxes in periods where output declines which limits their desire to issue public debt (this builds on the tax-smoothing literature of Barro (1979), Lucas and Stokey (1983), Bohn (1990)). In sum, both the quantity of public and private debt and their market yields are endogenous in our model.

The model leads to the three predictions. (1) A higher supply of public debt leads to a rise in the risk free rate (as in Bansal and Coleman (1996), and Krishnamurty and Jorgensen (2010)). A higher supply of public debt leads to a lower liquidity premium on Treasuries, which tends to raise its pecuniary return (i.e., the real risk free rate). (2) A higher supply of public debt leads to a lower supply of private debt. The role of financial intermediaries is to produce liquidity — they do this by selling liquid debt and investing the proceeds in risky non-liquidity providing assets. The liquid liabilities of financial intermediaries compete with government debt in providing liquidity services. A rise in government debt thus leads to a fall in private debt. (3) A rise in uncertainty leads to a fall in private sector debt and a rise in government sector debt. A potential cost to financial intermediaries in providing

liquidity is the risk they are exposed to by the consequent maturity and risk mismatch between their assets and liabilities. A rise in this uncertainty thus leads to a rise in the cost of providing liquidity, which leads financial intermediaries to providing fewer liquid assets. The Treasury does not perceive these same financial risks (e.g., bankruptcy risk) due to a rise in uncertainty, but instead perceives a rising cost stemming from a higher variability of distortionary tax rates. With a sufficiently high value of liquidity due to the contraction of private debt, the optimal response of the government is to expand its debt. Essentially, the treasury recognizes the role of liquidity services in the economy, so as the private sector shrinks its supply of debt and the liquidity premium rise, the treasury tends to issue more public debt even though the prospect of higher future tax-costs rise. If one associates such a fall in private liquid assets as a liquidity crunch, this is a model in which a rise in uncertainty leads to a liquidity crunch. We numerically solve our model to show these effects.

We use our model to empirically study the time-series behavior of public and private debt, the impact of their supplies on yields, and the role of macro economic uncertainty in determining the magnitude of debt and its yield. We highlight three key data-features in our empirical work. First, periods of high macro-economic volatility see a drop in private debt and an increase in government debt — this feature is a very robust data finding. Second, there is significant negative correlation in the supply of private and public debt . Third, the supply of short-term government debt affects their real yields—higher debt raises real rates (lowers prices).

We measure the amount of public debt (as a fraction of GDP) as those U.S. Treasury bills, notes, or bonds that have less than two years to maturity. The short-term and low volatility nature of these securities makes them an attractive option for fulfilling certain liability constraints of financial intermediaries. Therefore, they are likely to carry a liquidity premium. For robustness, we also experiment with an alternative measure of government debt as the overall level of U.S. Treasury debt across all maturities. Second, we measure short-term private sector debt (as a fraction of GDP) as open-market paper, the sum of commercial paper and bankers acceptances. As these private-sector securities are also short-term, highly-rated, and marketable, they may be issued to capture the liquidity premium. This measure is also employed in Greenwood, Hanson, and Stein (2010b). While we view open-market paper as a natural empirical counterpart to the private sector debt described in our model, we also consider an alternative measure of short-term private sector debt that incorporates, in addition to open-market paper, short-term bank loans following Greenwood, Hanson, and Stein (2010a).

Using the U.S. time series on the supply of government debt and short-term liquid assets produced by the private sector, as well as their corresponding rates of return, we test many of the predictions of the model, including those just mentioned. We find empirical support for all the key predictions of the model: an increase in government debt leads to an increase in the real return to government debt, an increase in government debt leads to a fall in the supply of private debt, and an increase in uncertainty leads to a fall in the supply of private debt and a rise in government debt. Using unconditional correlations and Vector Auto-regressions (VAR), we document that these findings are quite robust in the data for a post-war sample from 1950 to 2009. Some of these empirical dimensions are also featured in earlier research. Krishnamurthy and Vissing-Jorgensen (2010) document a negative correlation between the supply of government debt and the AAA - Treasury spread, but not real rates. Greenwood, Hanson, and Stein (2010a) document a negative dependence between the supply of government and private debt across the maturity spectrum as the private sector responds to shifts in the supply of government debt at various maturities. Section 2 presents the model and several examples. In section 3, we discuss the data employed in our empirical exercise. Section 4 presents the VAR methodology and key results. Finally, section 5 concludes.

2 The Model with no Uncertainty

Prior to developing the model, it will be important to highlight which aspects of an economy we are attempting to model. This will be particularly important when we match up variables in the model with empirical counterparts in the data.

To begin, consider an economy with four sectors: households, banks, other financial intermediaries, and firms (to be sure, the government is a fifth sector). From this perspective, a special role of banks is to provide checkable deposits to households. A special role of other financial intermediaries is to create short-term marketable securities. Table 1 offers a balance-sheet breakdown of a four-sector economy that is tailored to highlight a point regarding banks and other financial intermediaries that we wish to make.

There are a variety of papers, including Bansal and Coleman (1996), that explore the role of banks in offering checkable deposits that are backed up by purchases of various types of assets. This paper, in contrast, focuses on the role of other financial intermediaries in offering marketable short-term securities. To simplify matters, we will think of merging the household and banking sectors, and thereby consider a three-sector economy along the lines

Table 1: **Assets and Liabilities: Four Sectors**

households		banks		financial interm.		firms	
liabilities	assets	liabilities	assets	liabilities	assets	liabilities	assets
	equities gvt. debt pvt. debt bank dep.	bank dep.	equities gvt. debt pvt. debt	pvt. debt	equities	equities	

described in Table 2. Specifically, although we will refer to privately-supplied liquid assets, we do not have in mind checkable deposits, but rather short-term marketable securities created by the private sector.

Table 2: **Assets and Liabilities: Three Sectors**

households+banks		financial interm.		firms	
liabilities	assets	liabilities	assets	liabilities	assets
	equities gvt. debt pvt. debt	pvt. debt	equities	equities	

Consider a model with no uncertainty and two periods, an initial period and a terminal period. Households are endowed with one unit of time in each period and value consumption c over the two periods via the utility function

$$u(c) + \beta u(c'). \tag{1}$$

Households are simply assumed to begin the initial period with holdings of government bonds b, private debt d, and equity z_h in firms (expressed as a fraction of total equity). At the beginning of the period, they receive a payout on government bonds, a payout on private debt, and dividends from firms proportional to their equity holdings, yz_h (y is the dividend payout of firms and z_h is the fraction of the firm owned directly by households). As owners of the financial intermediaries households receive any financial intermediary profits Ω. During the period they supply n units of labor inelastically, receive wage payments

wn, pay a labor income tax τwn based on a proportional tax τ (in this version of the model with an inelastic labor supply, this will act like a lump-sum tax), receive a lump-sum government redistribution g, purchase consumption goods c, and choose new holdings of government bonds b' at price q_b, new private debt d' at price q_d, and new equity z'_h at price p_z. Purchasing consumption goods incurs a transaction cost $\varphi(c, b, d)$ that depends on consumption, holdings of government debt, and holdings of private debt. Assume that φ is homogenous of degree one in all three inputs, $\varphi > 0$, $\varphi_1 > 0$, $\varphi_2 < 0$, and that $\varphi_3 < 0$. In the terminal period the decision faced by households is similar, except they purchase no assets. Formally, households choose (c, b', d', z'_h, c') to maximize utility subject to the following budget constraints:

$$c + \varphi(c, b, d) + q_b b' + q_d d' + p_z(z'_h - z_h) = (1 - \tau)wn + g + yz_h + b + d + \Omega.$$

$$c' + \varphi(c', b', d') = (1 - \tau')w'n' + g' + y'z'_h + b' + d' + \Omega'.$$

Output is produced according to a constant-returns-to-scale production function f with a fixed factor k (also in fixed supply over time) and labor n: $af(k, n)$, where a is the level of total factor productivity, which for simplicity is assumed to be constant over time. Markets are assumed to be perfectly competitive, hence $wn = af_n(k, n)n$ is paid to workers as wage payments and the remainder $y = af_k(k, n)k$ is paid to owners of the firm. Firms are entirely owned by their equity holders, hence all non-wage payments are made to their equity holders.

The government issues government debt b', spends g, and levies a proportional tax τ on labor income. In doing so it must balance its budget constraint, which in flow form is given by

$$\tau af_n(k, n)n + q_b b' = b + g, \tag{2}$$

$$\tau' a' f_n(k, n')n' = b' + g. \tag{3}$$

For simplicity, it will be assumed that $b' = b$. The next section considers an optimal government debt policy.

Financial intermediaries issue private debt d' that provides a liquidity service as reflected in the transaction cost function φ, and use the proceeds to purchase equity z'_b. In this sense financial intermediaries transform illiquid assets into liquid assets. With uncertainty they would expose themselves to insolvency in performing such a task. The reward to a financial intermediary is the ability to exploit the liquidity premium by selling liquid assets and purchasing illiquid assets.

Financial intermediaries (potentially new ones) issue debt in the initial period in the amount of $q_d d'$ and purchase equity in the amount of $p_z z_b'$. By assumption, $p_z z_b' = q_d d'$. The debt incurs a promise to pay d' in the next period. The financial intermediary incurs a cost $\gamma p_z z_b'$, which is proportional to its size, for providing its services. This constant-returns-to-scale assumption means we do not need to be concerned about the size of individual banks. For simplicity assume that costs associated with a financial intermediary are lump-sum redistributed to households. Profits in the terminal period are equal to:

$$\Omega' = (y'/p_z - 1/q_d - \gamma) p_z z_b'. \tag{4}$$

In a competitive equilibrium, firms will earn zero discounted profits. Financial intermediaries, in the initial period, pay off outstanding debt and distribute profits to their shareholders (for the aggregate economy this division of revenue does not matter, so it will not be further specified). It will also be assumed that initial private debt d is such that $d' = d$. This assumption does matter, but it will be made to simplify the analysis in a way that would not seem to alter the qualitative properties of the model.

Define

$$M' = \frac{u_c(c')/(1 + \varphi_c(c', b', d'))}{u_c(c)/(1 + \varphi_c(c, b, d)}).$$

Note that, given the simplifying assumptions made, $M' = 1$.

The first-order conditions for households, firms, and financial intermediarires, after imposing market-clearing conditions, which includes a zero profit condition for financial intermediaries, and imposing assumptions such as $b' = b$, $d' = d$, and $c' = c$, are:

$$\begin{aligned} c + \varphi(c, b, d) &= af(k, n), & (5)\\ q_b &= \beta(1 - \varphi_b(c, b, d)), & (6)\\ q_d &= \beta(1 - \varphi_d(c, b, d)), & (7)\\ p_z &= \beta a f_k(k, n) k, & (8)\\ 0 &= \frac{a f_k(k, 1) k}{p_z} - \frac{1}{q_d} - \gamma. & (9) \end{aligned}$$

The scale of the banking sector as defined by d determines the split of the total return of private debt into a pecuniary and a non-pecuniary component, so the scale of the banking sector determines q_d. The zero-profit condition (9) essentially ensures that the cost of borrowing inclusive of interest and operating costs equals the return on the banks portfolio, so in this sense the zero-profit condition determines the scale of the banking sector. Also, in

equilibrium

$$z'_h + z'_b = 1,$$

and

$$p_z z_b = q_d d.$$

This system determines the unknowns $(c, q_b, q_d, p_z, z'_h, z'_b, d)$.

Given the solution for p_z, use eq. (9) to write q_d as

$$q_d = \frac{\beta}{1-\beta\gamma},$$

and use eq. (7) to write

$$\varphi_d(c, b, d) = \frac{-\beta\gamma}{1-\beta\gamma}.$$

This equation can be used to solve for d/c as a function of b/c.

To derive some sharp relationships, assume a Cobb-Douglas transaction cost function:

$$\varphi(c, b, d) = \bar{\varphi} c^{\alpha_1}(b)^{\alpha_2}(d)^{\alpha_3}, \tag{10}$$

with $\bar{\varphi} > 0$, $\alpha_1 + \alpha_2 + \alpha_3 = 1$, $\alpha_1 > 1$, $\alpha_2 < 0$, and $\alpha_3 < 0$. With the Cobb-Douglas transaction cost function, the relation just derived between d/c and b/c is given by

$$\frac{d}{c} = \left(\frac{\beta\gamma}{-\alpha_3\bar{\varphi}(1-\beta\gamma)}\right)^{\frac{1}{\alpha_3-1}} \left(\frac{b}{c}\right)^{\frac{-\alpha_2}{\alpha_3-1}}. \tag{11}$$

Re-write eq. (5) as

$$c\left(1+\varphi\left(1, \frac{b}{c}, \frac{d}{c}\right)\right) = af(k, 1). \tag{12}$$

Use eq. (11), and the functional form of φ, to write this as

$$c\left(1+\bar{\varphi}\left(\frac{\beta\gamma}{-\alpha_3\bar{\varphi}(1-\beta\gamma)}\right)^{\frac{\alpha_3}{\alpha_3-1}} \left(\frac{b}{c}\right)^{\frac{-\alpha_2}{\alpha_3-1}}\right) = af(k, 1). \tag{13}$$

Note that the left side of this equation is a strictly-increasing function of c, and that there exists a solution c such that $0 < c < af(k, 1)$. Eq. (13) thus determines c. With c determined, d is determined by eq. (11). The bond price function q_b is determined by eq. (6), which can be written as

$$q_b = \beta\left(1-\alpha_2\bar{\varphi}\left(\frac{\beta\gamma}{-\alpha_3\bar{\varphi}(1-\beta\gamma)}\right)^{\frac{\alpha_3}{\alpha_3-1}} \left(\frac{b}{c}\right)^{\frac{\alpha_1}{\alpha_3-1}}\right). \tag{14}$$

These results determine the entire equilibrium.

Let's now derive some qualitative results. Eq. (11) shows that a rise in b/c is associated with a fall in d/c. That is, a rise in government debt as a fraction of consumption leads to a fall in private debt as a fraction of consumption. From eq. (13) it follows that a rise in b leads to a rise in b/c: if not, then the left side of eq. (13) will exceed $af(k,1)$ following the rise in b, which is a contradiction. Taken together, this means that a rise in government debt will lead to a rise in government debt relative to GDP and a fall in private debt relative to GDP. Essentially, a rise in government debt reduces the transaction-service return of private debt, which is thus met by a reduction in the amount of private debt. This is the substitution effect between government and private liquid assets that was described in the introduction. This effect is described graphically in Fig. 1.[1]

Note that a rise in b/c will not affect q_d, as q_d is given by the zero profit condition for financial intermediaries, which is unaffected by b/c. However, a rise in b/c will affect q_b. From eq. (14) it follows that a rise in b, and hence a rise in b/c, leads to a fall in q_b. Essentially, a rise in government debt reduces the transaction-service return to government debt, hence leading to a rise in the pecuniary return to government debt. This effect is described graphically in Fig. 2.[2] Consequently, a rise in b leads to a fall in the spread between interest rates on d and b. This is the liquidity premium effect of changes in the supply of government debt that was described in the introduction.

To summarize, a rise in government debt will lead to: (1) a fall in private debt relative to GDP, (2) a rise in the yield in government debt, and (3) and fall in the spread between the yield on private debt and government debt.

3 The Model with Uncertainty

This section introduces uncertainty into the model in order to study two issues. First, this section will study how a rise in uncertainty affects the supply of private liquid assets by financial intermediaries. Second, this section will consider an optimal government policy with regard to managing the supply of government liquid assets. The role of uncertainty in studying an optimal government policy will be important, as tax smoothing will be a key margin we will consider in studying the costs of varying levels of government liquid assets.

[1] The parameter values that generated this figure are $\alpha_1 = 1.5$, $\alpha_2 = -.25$, $\alpha_3 = -.25$, $\gamma = .02$, and $\beta = .9$.

[2] The parameter values that generated this figure are the same as those for Fig. 1.

As a tax-smoothing margin relies on distortionary taxes (following Barro (1979), Lucas and Stokey (1983), Bohn (1990)), this section will also introduce an elastic labor-leisure choice.

Many of the model's features are the same as before, so only new features will be described here. Households are endowed with one unit of time in each of two periods, and value consumption c and leisure $\ell = 1 - n$ over the two periods via the expected utility function

$$u(c, 1-n) + \beta E\{u(c', 1-n')\}. \tag{15}$$

Due to the possibility of default, private debt has a random payout x per unit of private debt. Households now choose $(c, n, b', d', z_h', c', n')$, where (c', n') are state-contingent functions of information revealed in the terminal period, to maximize expected utility subject to the following budget constraints:

$$c + \varphi(c, b, d) + q_b b' + q_d d' + p_z(z_h' - z_h) = (1-\tau)wn + g + yz_h + b + xd + \Omega,$$

$$c' + \varphi(c', b', d') = (1-\tau')w'n' + g' + y'z_h' + b' + x'd' + \Omega'.$$

In terms of output, the fixed fixed factor k is again assumed to be in fixed supply over time, but the level a of total factor productivity is stochastic. Productivity in the terminal period, a', is not known in the initial period.

The government pursues a debt/taxation policy to maximize utility of the representative household. If debt could be issued without cost, clearly it would be optimal for the government to issue unlimited amounts of debt. To limit the amount of debt the government chooses to issue, there must be a cost associated with issuing debt. The cost explored here stems from tax smoothing. That is, the more government debt that is issued in the initial period, the greater is the variability in tax rates to pay off the debt in the terminal period. Thus, more government debt leads to a higher uncertainty regarding the level of tax rates in future periods. This higher level of uncertainty regarding future tax rates is balanced off by the liquidity benefits from issuing debt, thereby leading to an optimal amount of debt.

To capture this margin determining government actions, the government is modeled in the following way. Each period the government must finance an exogenous level of government spending g by levying distortionary taxes on labor income as well as issuing debt. Any excess of revenue over spending is invested by purchasing equity in the firms in the economy. In this sense, more government debt issued leads to a greater share of private equity held by the government. More holdings of private equity leads to a greater variability of tax rates in the future, as variability in future equity values leads to a corresponding variability in the fraction of government spending being financed by selling equities.

The reward to a financial intermediary from issuing private debt is the ability to exploit the liquidity premium by selling liquid assets and purchasing illiquid assets. The cost, with uncertainty, is the risk of insolvency. The greater the uncertainty in the economy, the less willing financial intermediaries are to create liquid assets. In equilibrium the benefit of liquidity is balanced against the cost of bankruptcy. In the event that the financial intermediary is unable make full payment, it pays out all remaining revenue to the debt holders. In addition to the cost of providing its services, the financial intermediary also incurs a cost $\xi p_z z_b'$ in the event of bankruptcy; this cost is also proportional to its size. It is assumed that the bankruptcy cost is borne directly by the owners/managers of the financial intermediary. For simplicity assume that costs associated with a financial intermediary are lump-sum redistributed to households. Profits in the terminal period are equal to:

$$\Omega' = \begin{cases} (y'/p_z - 1/q_d - \gamma) p_z z_b' & \text{if } y'/p_z - 1/q_d - \gamma \geq 0 \\ -\xi p_z z_b' & \text{if } y'/p_z - 1/q_d - \gamma < 0 \end{cases} \tag{16}$$

and, since $p_z z_b' = q_d d'$, the payout rate on debt d' is given by

$$x' = \min\{1, (y'/p_z - \gamma) q_d\}. \tag{17}$$

Financial intermediaries, in the initial period, pay off outstanding debt and distribute profits to their shareholders (for the aggregate economy this division of revenue does not matter, so it will not be further specified).

Re-define

$$M' = \frac{u_c(c', 1-n')/\varphi_c(c', b', d')}{u_c(c, 1-n)/\varphi_c(c, b, d)}.$$

The first-order conditions for households, firms, and financial intermediarires, after imposing market-clearing conditions, are:

$$c + \varphi(c, b, d) = af(k, n), \tag{18}$$

$$c' + \varphi(c', b', d') = a'f(k, n'), \tag{19}$$

$$\frac{u_\ell(c, 1-n)}{u_c(c, 1-n)} = \frac{(1-\tau) a f_n(k, n)}{1 + \varphi_c(c, b, d)}, \tag{20}$$

$$\frac{u_\ell(c', 1-n')}{u_c(c', 1-n')} = \frac{(1-\tau') a' f_n(k, n')}{1 + \varphi_c(c', b', d')}, \tag{21}$$

$$q_b = \beta E[M'(1 - \varphi_b(c', b', d'))], \tag{22}$$

$$q_d = \beta E[M'(x' - \varphi_d(c', b', d'))], \tag{23}$$

$$p_z = \beta E[M' a' f_k(k, n') k]. \tag{24}$$

In equilibrium, expected discounted financial intermediary profits must equal zero:

$$E[M'\Omega'] = 0.$$

Also, in equilibrium

$$z_h' + z_b' + z_g' = 1,$$

and

$$p_z z_b' = q_d d'.$$

The government can choose any policy (τ, b', z_g', τ'), where τ' is a state contingent function of a', that is feasible. A feasible policy is one that balances the government's budget:

$$\tau a f_n(k,n)n + a f_k(k,n)k z_g + q_b b' + p_z(z_g - z_g') = b + g, \quad (25)$$

$$\tau' a' f_n(k,n')n' + a' f_k(k,n')k z_g' = b' + g. \quad (26)$$

An optimal government policy is one that maximize overall household utility subject to the market equilibrium conditions and feasibility.

This system determines the unknowns $(c, n, q_b, q_d, p_z, z_h', z_d', z_g', \tau, b', d', c', n', \tau')$, with (c', n', τ') as functions of a'.

To demonstrate the response of the private and government sector supply of liquid assets to a rise in uncertainty, we will compute a numerical solution to the model and run an experiment in which uncertainty rises.

Numerical solutions to this model can be obtained in the following manner. First, fix (τ, b', z_g'). Use eqs. (19), (21), and (26) to solve for $(c', n'\tau')$ as functions of (b', z_g', d'). That is, solve for the policy functions in the terminal period. Use these functions and remaining first-order/equilibrium conditions to solve for $(c, n, q_b, q_d, p_z, z_h', z_d', \tau, d', c', n', \tau')$. A simple Newton's algorithm seems to perform well for this problem. This provides a solution to the first-order/equilibrium conditions given government policy choices b' and z_g'. Finally, search over b' and z_g' to maximize the expected utility of households, given the dependence of the equilibrium on these choices.

In solving this model numerically, the following functional forms were chosen. Utility was chosen to be:

$$u(c, 1-n) = \log(c) + \log(1-n). \quad (27)$$

The transaction cost function is given by eq. (10). The production function is given by:

$$f(k,n) = k^\sigma n^{1-\sigma}. \quad (28)$$

Productivity a in the initial period is normalized to $a_0 = 1$, but can take on two values in the terminal period, a_L and a_H, with probabilities π_L and π_H.

Specific values of parameters, while guided by parameter choices made in the literature and other quantitative observations on the post-war U.S. economy, are largely taken for purposes of illustration to demonstrate that this model can indeed exhibit what we are referring to as an uncertainty effect. Capital is simply initialized to 1 in both periods, $\beta = .9$, and $\sigma = .3$. The level of productivity is also initialized to one in the initial period, $a_0 = 1$, but can take one two values in the terminal period, a_L and a_H with probabilities π_L and π_H. For the benchmark model, $a_L = .95$, $a_H = 1.05$, $\pi_L = .5$, and $\pi_H = .5$. Both government and private debt in the initial period are chosen to be about 20 percent of GDP, and the government is assumed to own 10 percent of outstanding shares in the economy. Government spending is assumed to be about 10 percent of GDP in both periods. The transaction cost function parameters are the same as what we chose for the deterministic version of this model (note that the transaction cost function scale parameter $\bar{\varphi} = .01$ was chosen such that transactions cost as a fraction of GDP is about 2 percent). The remaining parameters γ and ξ were chosen to achieve a return on short-term government debt of about $r'_b = 5$ percent and a yield spread between short-term private and government debt also about $r'_d - r'_b = 4$ percent. The values of all parameters are reported in Table 1.

Let's examine, now, the ability of this model to capture the uncertainty effect. The uncertainty concerns the likelihood of bankruptcy, so the experiment that needs to be performed is one in which the probability of bankruptcy rises. To do so, this paper will report simulations of an experiment in which uncertainty in the economy rises due to a rise in the probability of a bad event. Specifically, the experiment considered is a rise in π_L (and a consequent fall in π_H). Fig. 3 exhibits the response of private liquid assets to a rise in π_L from .5 to .6, and Fig. 4 exhibits the response of government liquid assets to such a rise in the probability of a bad event. Note that the supply of private liquid assets falls and the supply of government liquid assets rises. These results seems straightforward to interpret. As uncertainty rises regarding the risk of default, financial intermediaries find it more risky to issue debt, consequently they issue less. A contraction in private debt leads to a rise in marginal value of government debt, and hence leads to an expansion of government debt. The government, however, does not wish to issue too much debt, as issuing more debt leads to more variability in tax rates in the terminal period. In choosing the optimal response, the government balances off the benefit of increasing liquidity against the cost of financing this liquidity with variable distortionary tax rates. The net result is reflected in an uncertainty

Table 3: **Parameter Values**

parameter	*value*
β	.9
γ	.02
ξ	.05
$\bar{\varphi}$	.01
α_1	1.50
α_2	-.25
α_3	-.25
σ	.30
a_0	1.00
a_L	.95
a_H	1.05
k	1.00
g	.05
b	.10
d	.10
z_g	.10

effect on liquidity that was mentioned in the introduction.

4 Data Description

We turn to an exploration of the empirical relations shared among government and private debt quantities and relative prices using post-war data spanning the first quarter of 1950 to the first quarter of 2009. In our formal VAR estimation, we focus on the dynamic process jointly governing variation in debt quantities and prices (yields). To measure the variation of government and private debt quantities through time, we construct ratios of these amounts relative to GDP. Real U.S. GDP is obtained from the National Income and Product Accounts at the Bureau of Economic Analysis, and we include the growth rate in real GDP as a control

variable in all our regressions.[3]

First, we measure the amount of government debt (relative to GDP). For our main measure of government debt, we focus on those bills, notes, or bonds in the CRSP bond database that have less than 2-years maturity. The short-term and low volatility nature of these Treasury securities makes them an attractive option for fulfilling certain liability constraints of financial intermediaries. Therefore, they are likely to carry a liquidity premium. For robustness, we also consider an alternative measure of government debt as the overall level of U.S. Treasury debt (of any maturity) scaled by GDP by including all bonds covered in CRSP.

Second, we measure short-term private sector debt (relative to GDP) from the Federal Reserve's *Flow of Funds* accounts, which tracks financial flows throughout the U.S. economy. As described in the integrated balance sheet in Table 2, this paper's focus is on the role of other financial intermediaries in offering marketable short-term securities. To measure private securities that may carry a liquidity premium, we define short-term private debt as open-market paper (Table F.208). Open-market paper includes commercial paper and bankers acceptances associated with both the domestic financial and non-farm, non-financial corporate sectors. As these private-sector securities are short-term, highly-rated, and marketable, they may be issued to capture the liquidity premium. This measure is also employed in Greenwood, Hanson, and Stein (2010b). While we view open-market paper as a natural empirical counterpart to the private sector debt described in our model, we also consider an alternative measure of short-term private sector debt that incorporates, in addition to open-market paper, short-term bank loans following Greenwood, Hanson, and Stein (2010a). This alternative measure is the sum of quarterly observations on open-market paper (Table F.208), bank loans not elsewhere classified (Table F.215), and other loans and advances (Table F.216).

For prices (yields), we consider several alternatives. First, we measure real Treasury bill rates as well as several relevant yield spreads. For the post-war sample, the real Treasury bill rate is computed as the 3-month Treasury bill rate, obtained from the Federal Reserve's

[3]Our private and government debt measures are based on accounting values (market values at issuance) rather than current market values. Given that we focus largely on short-term private and government issuance, this is not likely a significant issue. Further, Hall (2001) constructs a market value series from the Flow of Funds accounts for a subset of these data. Using his data where available, constructed debt to GDP ratios, where debt is measured either as the accounting or market value, are very highly correlated.

release on interest rates (H.15), less a measure of expected inflation.[4] To measure expected inflation, we use the year-on-year percentage change in the GDP deflator, lagged one quarter to ensure that the information is known. To explore robustness to the measurement of the real yield, we also consider several alternative measures in the price dimension. In particular, we include the yield spread between AAA rated bonds and Treasury bonds of similar maturity. Employing the spread allows us to avoid the measurement of expected inflation. Krishnamurthy and Vissing-Jorgensen (2008) also employ the AAA - Treasury spread. In unreported results, we also consider the yield spread between BAA rated bonds and Treasury bonds of similar maturity and commercial paper and Treasury bills of similar maturity; the evidence is very similar. All necessary data items for the construction of these yield spreads are obtained from the Federal Reserve's release on interest rates (H.15).

Finally, we construct measures macroeconomic volatility and risk compensation to explore the relationships among these various quantities, prices, and levels of economic uncertainty. First, we compute conditional GDP growth volatility based on a GARCH process for the real quarterly GDP growth rate. Second, we also construct an estimated measure of the equity market risk premium. More precisely, the expected market risk premium is the fitted process implied by the following standard return predictability regression:

$$Ret_{mkt,t+1} = \alpha_0 + \alpha_1 \text{MktDividendYield}_t + \alpha_2 \text{TermSpread}_t + \alpha_3 \text{TbillRate}_t + \epsilon_{t+1} \quad (29)$$

where $Ret_{mkt,t+1}$ is the quarterly excess return on the CRSP market portfolio, $\text{MktDividendYield}_t$ is the dividend yield on the market portfolio, TermSpread_t is the term spread between long-run and short-run U.S. government bonds, and TbillRate_t is the 3-month Treasury bill rate. The regression results are provided in Table 5. While the predictive R^2 of this regression is only 0.043, it is generally consistent with the existing literature on return predictability. The regression suggests an important role for the lagged dividend yield, which is also consistent with the previous literature. For the remainder of the paper, we will use this constructed series to directly capture risk compensation associated with economic uncertainty.

For the post-war sample, summary statistics for each variable are provided in Table 4. To gauge the importance of our private debt measure, note that open-market paper is about 6% of GDP on average. While the ratio is not huge, it has grown consistently over time, exceeding 10% in the past decade. More importantly, open-market paper is consistently the single largest asset class held by money market mutual funds (see Table F.121 of the Flow

[4] We compute the real Treasury bill rate as $\frac{(1+r_f)}{(1+\pi)} - 1$, where r_f is the nominal 3-month Treasury bill rate and π is our simple measure of expected inflation.

of Funds accounts), well in excess of their holdings of U.S. Treasury securities. Since money market mutual funds are highly constrained in what securities they may hold in terms of maturity and risk profile, it seems private sector open-market paper is competing to provide liquidity services.

One other aspect of the data that should be immediately acknowledged is the high level of persistence of several of the debt series. The auto-correlation exceeds 0.99 for the ratio of private debt to GDP; for visual inspection, debt quantities across the full sample are presented graphically in Figure 5. In our main empirical exercises (presented in the next section), we employ a Vector Auto-regression (VAR) to account for the persistence in these series as well as to provide a methodology to analyze independent variation across the various quantities of interest as opposed to simply documenting unconditional correlations.

Before moving the formal VAR estimated over the full post-war sample, however, we also provide some casual evidence on the our main quantities of interest, permitting an exploration of the shared relations among these variables over several business cycles. Table 6 provides simple unconditional correlations across the debt quantities (the logged ratio, relative to GDP) and prices over the post-war period. Several features of the data are worth highlighting. First, an increase in government debt in the model reduces the transaction-service return to government debt and private debt, resulting in a reduction in the relative amount of private debt. The correlation across the relative government and private debt measures are significantly negative, consistent with the implications of our model as presented above. For example, the correlation between short-term government debt and open-market paper (private debt) is -0.31. This inverse relationship is further shown graphically in Figure 6. Second, an increase in government debt in the model is also associated with a reduction in the spread between interest rates on private and government debt. The AAA-Treasury spread is inversely related to total government debt, with a correlation of -0.30, but positively related to private debt with a correlation of 0.52. Last, an elevated level of uncertainty makes it potentially more costly for banks to provide liquidity services. Economic uncertainty, as measured by our market risk premium variable, is associated with a lower level of relative private debt, with a correlation of -0.75. All these features of the data, including those for our alternative debt measures, are in-line with the predictions of the model. Next, we turn to the formal VAR analysis on the post-war data.

5 Vector Auto-regression

To explore the dynamic features of the quantities and prices implied by the model, we estimate several vector auto-regressions (VARs) based on quarterly data. Employing a VAR structure has two advantages. First, we can directly deal with the extreme level of persistence exhibited by several of our series. Second, the VAR provides a framework for evaluating the correlations among the relevant independent (orthogonal) shocks to the system and their impact on the key variables of interest. We will primarily evaluate the latter through estimated impulse response functions.

We consider several alternative VAR representations of the data, including a VAR(4) [four-quarters], VAR(8) [eight-quarters], and a more parsimonious version that incorporates only the first quarter's, the first year's, and the second year's lag terms. The last specification is the one we will focus on since the lags associated with these particular periods appear to be the most important and presentation of the more parsimonious version is less cluttered. Nevertheless, these alternative specifications provide comparable empirical results. In particular, the impulse responses functions to which we will pay particular attention are largely unchanged. The variables in our VAR system are (1) the real GDP growth rate, (2) the (estimated) market risk premium, (1) the log of the government debt to GDP ratio, (4) the log of the private sector debt to GDP ratio, and (5) the real Treasury bill rate. For the impulse response functions, we will retain this ordering as the first two variables of interest are exogenous to the model (and the associated simulations presented in an earlier section). We are interested in the response of final two endogenous variables to shocks in the supply of government debt and levels of economic uncertainty. However, we do place the government debt to GDP ratio after the GDP growth rate and our measure of uncertainty. While we are primarily interested in exploring the reaction of private debt and associated prices to changes in the amount of public debt, we must acknowledge that active policy may confound that exploration. To the extent that public policy makers (optimally) react to shocks associated with growth or uncertainty by changing public debt levels, we want to correctly attribute a movement in government debt levels to those deeper stimuli before then judging the degree to which private debt levels or market prices are affected by changes in the amount of public debt. In essence, by placing relative government debt levels third in our ordering, we will isolate the degree to which private debt levels and Treasury bill rates respond after controlling for deeper growth and uncertainty shocks.

Table 7 presents estimates for our baseline VAR. Several key results are worth noting.

First, as mentioned, the series are generally quite persistent, as is evidenced by the large and highly significant auto-regressive coefficients associated with each series (other than GDP growth). This suggests that taking account of the dynamic structure of these data is very important for exploring the role for unexpected variation in each. The standard error associated with each variable in the system implies that the variability of the unexpected shocks are much smaller than the overall level of each variable. For comparison, the unconditional standard deviations for each variable are provided in Table 7. Second, the R^2 associated with each variable in the system are large. While we are able to capture most of the temporal variation, this is almost certainly due in large part to the highly persistent nature of each series (again, excluding GDP growth). Third, there are important cross-predictability effects. In particular, the market risk premium effects suggest an important role for economic uncertainty.

As mentioned, we place the government debt variable third in the VAR ordering given that policy makers may optimally react to the economic conditions they face. In line with the model, an examination of the relationships between government and private debt levels should account for the fact that changes in government debt may indeed reflect a response to the macroeconomic environment. To properly explore the relationship between government and private debt activity, we need to first account for these features of the data. Figure 4 presents two impulse responses associated with the reaction of the log government debt ratio to either GDP growth or market uncertainty shocks. The log government debt level responds positively (and persistently) to a GDP growth shock and negative (and persistently) to an uncertainty shock.[5]

To evaluate the dynamic relations between our variables, Figure 5 provides a set of *four* impulse response functions of particular interest based on the estimated VAR. To explore the predictions of our model detailed above, we focus exclusively on the responses of (1) the log of the short-term private sector debt to GDP ratio and (2) the short-term real Treasury bill rate to one-standard deviation impulses in (1) the log of short-term government debt to GDP ratio and (2) the estimated market risk premium. These responses describe the manner in which the private sector responds to unexpected shocks in either the supply of government debt or economic uncertainty, controlling for the fact that the government debt levels themselves are responding to the macroeconomic environment. We also provide 95% confidence intervals around each impulse response.

[5] In these pictures, we employ the main short-term government debt ratio; however, the responses are similar for the total government debt ratio as well.

Several features deserve attention. First, as predicted by the model, private sector debt falls in a statistically significant manner in response to a positive shock to government debt. The magnitude is economically meaningful as well. An unexpected (one standard deviation) increase in the government debt ratio of about 5% engenders a decline in relative private sector debt of about 1.7%. The negative response in the log private debt ratio is at its largest point (in absolute magnitude) after about eight quarters. Second, the response of the private debt ratio to economic uncertainty is even more pronounced. As predicted by the model an unexpected increase in economic uncertainty, as measured by an increase in the market risk premium of about 200 basis points, yields a decline in the ratio of short-term private sector debt of about 4% after 8 quarters. This response of private sector debt to unexpected changes in economic uncertainty persists for several years.

We also report the responses of the real Treasury bill rate to shocks in either the government debt ratio or economic uncertainty. Both effects are statistically significant, but the responses are not as significant in economic terms as the responses documented for private debt levels. In response to an unexpected change in the ratio of government debt to GDP of about 5%, the real Treasury bill rate increases by about 20 basis points. The responses of the real T-bill rate to a shock in economic uncertainty is somewhat more economically significant. An one-standard deviation increase in the market risk premium engenders a decline in the real T-bill rate of about 70 basis points.

Taken together, the estimated VARs and the particular impulse responses we highlight are largely in line with the model's predictions. That said, one important issue requires some attention. In some cases, we observe a delayed response to the various shocks we consider. The model has no role for these kinds of dynamics. A more involved model with additional frictions or costs could potentially deliver this kind of temporal dependency, but that detail is beyond the aims of the current paper. Rather, acknowledging this issue, we want to demonstrate that the data are largely in line with the implications of a fairly simple model of aggregate liquidity provision.

5.1 Alternatives Measures

To explore the robustness of our long-history results, we also consider several cases where we replace key variables with plausible alternatives that may capture the relevant components implied by the model. In each case, we estimate the parsimonious VAR, but we replace either the debt quantity or price measure with a reasonable alternative. In the interests of

space, we do not report the VAR estimates nor all the impulse responses, but rather we plot two example sets of the four particular impulse response functions of interest implied by the estimated VARs. As above, these are the responses of (1) the relevant private debt quantity measure and (2) the relevant price (yield) measures to shocks in (1) the government debt measure and (2) the estimated market risk premium, where each case considers alternatives for each of these.

First, we consider the following alternatives along the debt quantity dimension where we replace the log of short-term government debt to GDP ratio with the overall government debt to GDP ratio (all maturities). The aggregate amount of government debt may be a reasonable alternative as the full maturity spectrum of Treasury bonds is potentially important in aggregate liquidity provision. As before, we place the government debt ratio after the primary macroeconomic shocks to control for the degree to which policy makers may react to these stimuli. Figure 6 shows that the impulse response functions are quite similar to that presented above, and, in fact, suggest that the negative reaction of the private debt levels to an increase in *overall* government debt is somewhat stronger. We also consider an alternative where we include the broader private credit to GDP measure from Greenwood, Hanson, and Stein (2010a); as the results are nearly identical, we exclude them, but they are available upon request.

Second, we consider the following alternatives along the price (yield) dimension where we replace the real Treasury bill rate with the AAA spread. The AAA (relative to comparable Treasuries) provides some gauge of the relative pricing of government and private debt, and also avoids the difficult direct measurement of expected inflation (see Krishnamurthy and Vissing-Jorgensen (2008)). The AAA spreads describe long-dated debt instruments (though the yields there are highly correlated with near to medium term issues of similarly rated debt). Our view is that the extent to which the evidence is robust across an alternative choices only bolsters our claim that the data are largely in-line with the predictions of the model. As can be seen in Figure 6, the debt quantity responses are nearly identical (as you might expect); however, the yield spread response now moves in the opposite direction. This is as the model simulations predict. The yield spread response to an increase in government debt (relative to GDP) is negative, potentially reflecting a diminished liquidity premium of lower cost (in our model) government debt relative to the private alternatives. As with many of the other responses, though, the reaction is again delayed. Finally, the yield spread does increases significantly (and swiftly) with market uncertainty, potentially reflecting the increased probability of insolvency among issuers. In unreported results, we also considered

two additional spreads: (1) the BAA spread relative to comparable Treasuries and (2) the commercial paper spread relative to comparable Treasuries. In either case, the evidence is also largely in-line with the results presented above.

6 Conclusions

We present a model which helps understand the links between the supply of government debt, the supply of private debt, the liquidity premium, and uncertainty. In particular, the model endogenizes the supply of private debt and captures three key features: (i) a higher supply of government debt lowers the liquidity premium in Treasury-bills and hence raises the real risk free rate; (ii) higher levels of government debt lower the supply of private debt as the incentives of the private sector to capture the liquidity premia diminish; and (iii) an increase in economic uncertainty raises the insolvency costs for intermediaries and hence also lowers the supply of short-term private debt. Using post-war U.S. data, we show that these implications have strong empirical support. Our quantitative and empirical analysis suggests that episodes of a liquidity crisis which exhibit sharp declines in issuance of commercial paper and other short-term private securities reflect the forces featured in the model — higher aggregate risk and an increased supply of short-term government debt. That is, financial intermediaries facing higher levels of uncertainty optimally choose to reduce their borrowing and lending activities as their ability to capture the liquidity premium has to be traded-off against the increased cost of insolvency.

7 References

Bansal, R. and W.J. Coleman, 1996. "A Monetary Explanation of the Equity Premium, Term Premium, and Risk-Free Rate Puzzles," *Journal of Political Economy* 104, 1135-1171.

Barro, R., 1979, On the Determination of the Public Debt, *Journal of Political Economy*, 87, 940-971.

Bohn, H., 1990, Tax Smoothing with Financial Instruments, *American Economic Review*, 80, 1217-1230.

Greenwood, R., S. Hanson, and J. Stein, 2010a, "A Gap-Filling Theory of Corporate Debt Maturity Choice," *Journal of Finance*, 65, 993-1028.

Greenwood, R., S. Hanson, and J. Stein, 2010b, "A Comparative-Advantage Approach to Government Debt Maturity," working paper, Harvard University.

Gurley, J. G., and E. S. Shaw, 1955, "Financial Aspects of Economic Development," *American Economic Review*, 45(4), 515-538.

Friedman, M., 1969, "The Optimum Quantity of Money," In *The Optimum Quantity of Money and Other Essays*, Chicago: Aldine.

Hall, R., 2001, "The Stock Market and Capital Accumulation," *American Economic Review* 91, 1185-1202.

Krishnamurthy, A. and A. Vissing-Jorgensen, 2008, "The Demand for Treasury Debt," working paper, Northwestern University.

Lucas, R. E., and N. Stokey, 1983, "Optimal fiscal and monetary policy in an economy without capital," *Journal of Monetary Economics*, 12, 55-93.

Lucas, R. E., and N. Stokey, 1987, "Money and Interest in a Cash-in-Advance Economy," *Econometrica*, 55, 491-513.

Marshall, D., 1992, "Inflation and Asset Returns in a Monetary Economy," *Journal of Finance* 47, 1315-1342.

Tobin, J., 1963, "An Essay on Principles of Debt Management." In *Fiscal and Debt Manage-*

ment Policies, by William Fellner et al. Englewood Cliffs, N.J.: Prentice- Hall (for Comm. Money and Credit).

Townsend, T.,1987, “Asset-Return Anomalies in a Monetary Economy,” *Journal of Economic Theory* 41, 219-247.

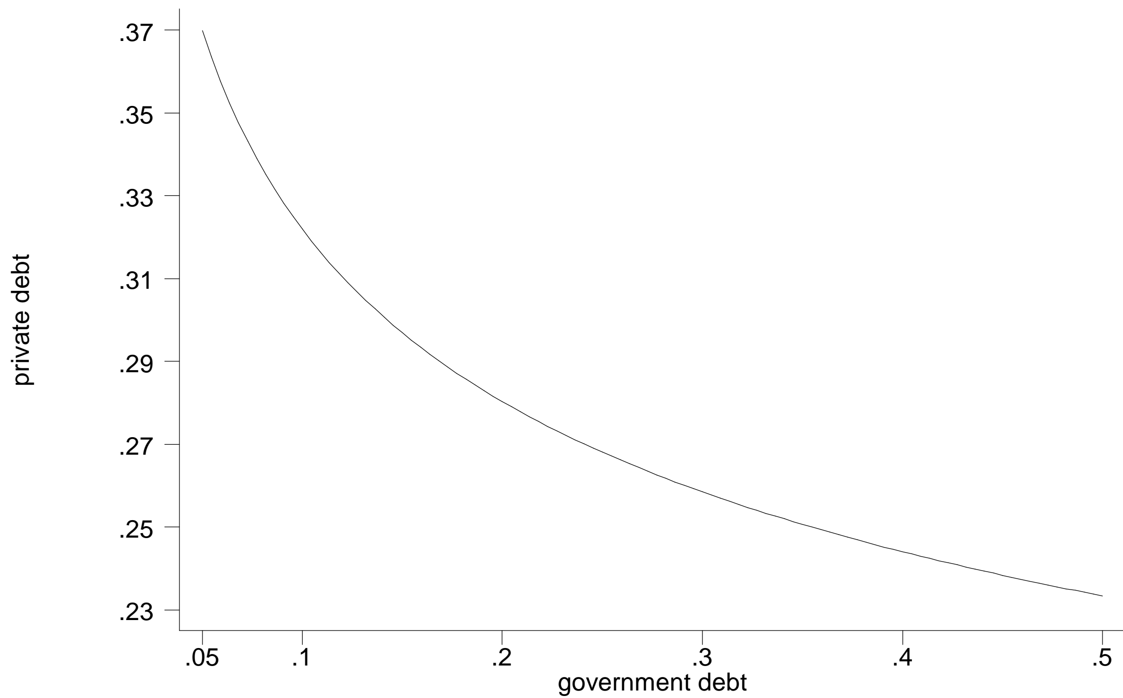

Fig 1. The Substitution Effect

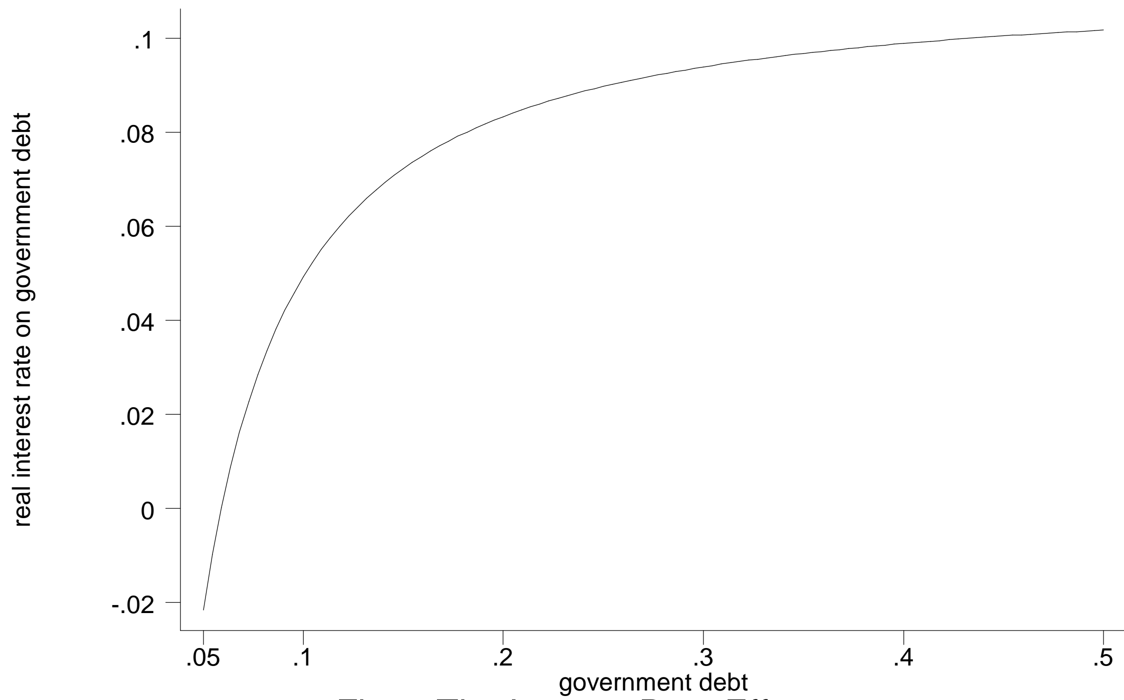

Fig 2. The Interest Rate Effect

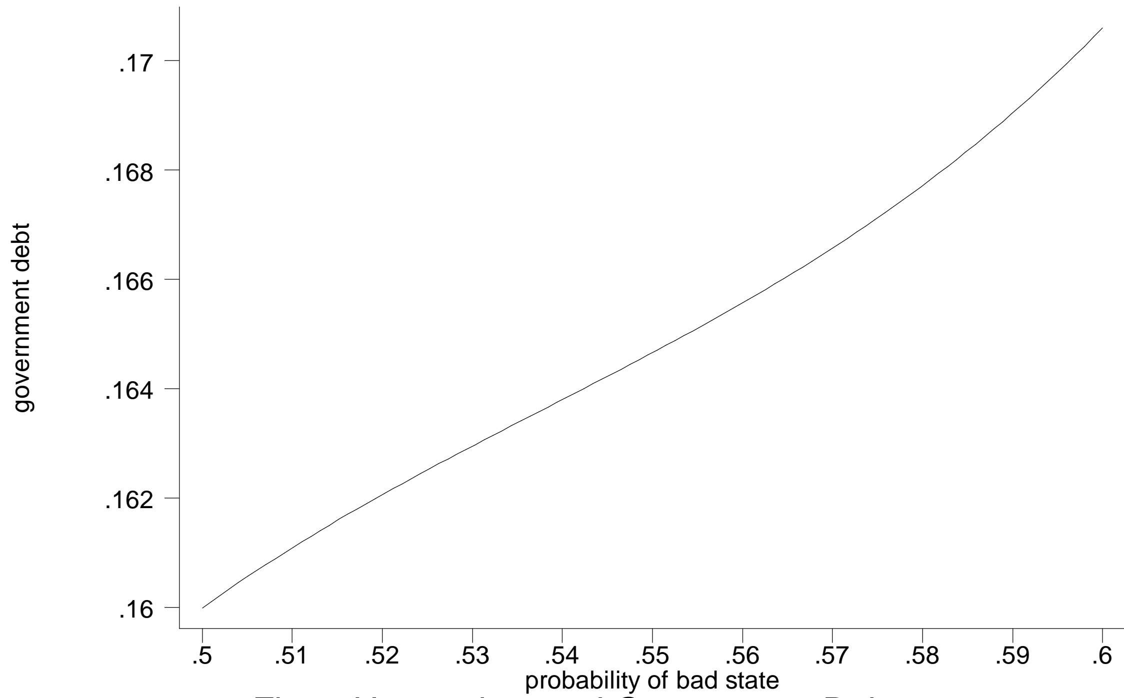

Fig 3. Uncertainty and Government Debt

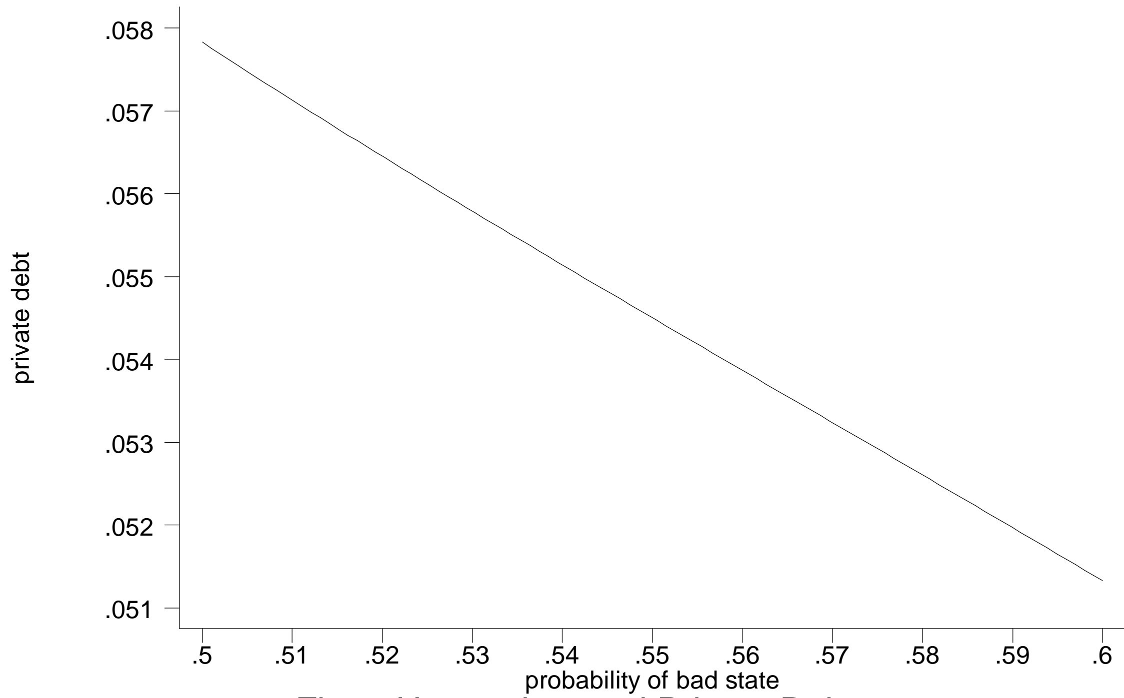

Fig 4. Uncertainty and Private Debt

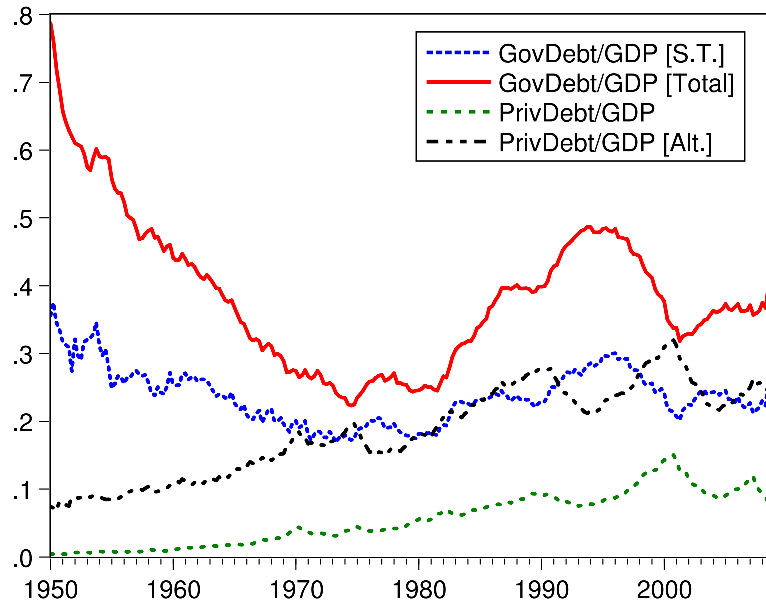

Figure 5: **Quantities (Ratios) 1950-2009** This figure provides quarterly observations from 1950-2009 on our measures of the quantitiy of debt (as a fraction of GDP). For public debt, we report two measures: (1) GovDebt/GDP [S.T.] represents those bills, notes, or bonds in the CRSP bond database that have less than 2-years maturity and (2) GovDebt/GDP [Total] represents the overall level of U.S. Treasury debt (of any maturity) by including all bonds covered in CRSP. For private debt, we also report two measures: (1) PrivDebt/GDP [S.T.] represents open-market paper and (2) PrivDebt/GDP [Alt.], following Greenwood, Hanson, and Stein (2010a), represents the sum of quarterly observations on open-market paper, bank loans not elsewhere classified, and other loans and advances.

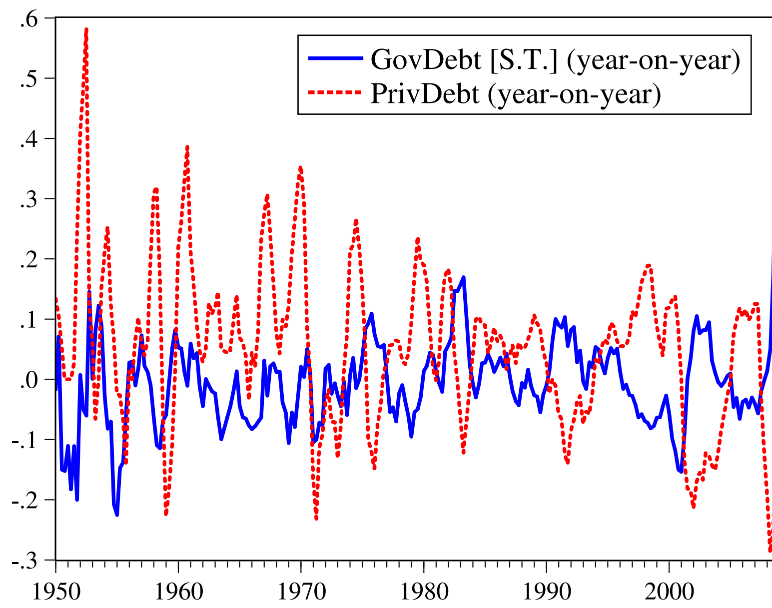

Figure 6: **Quantities (Growth) 1950-2009** This figure provides year-on-year real growth in private debt (open market paper) and public debt (short-term treasuries).

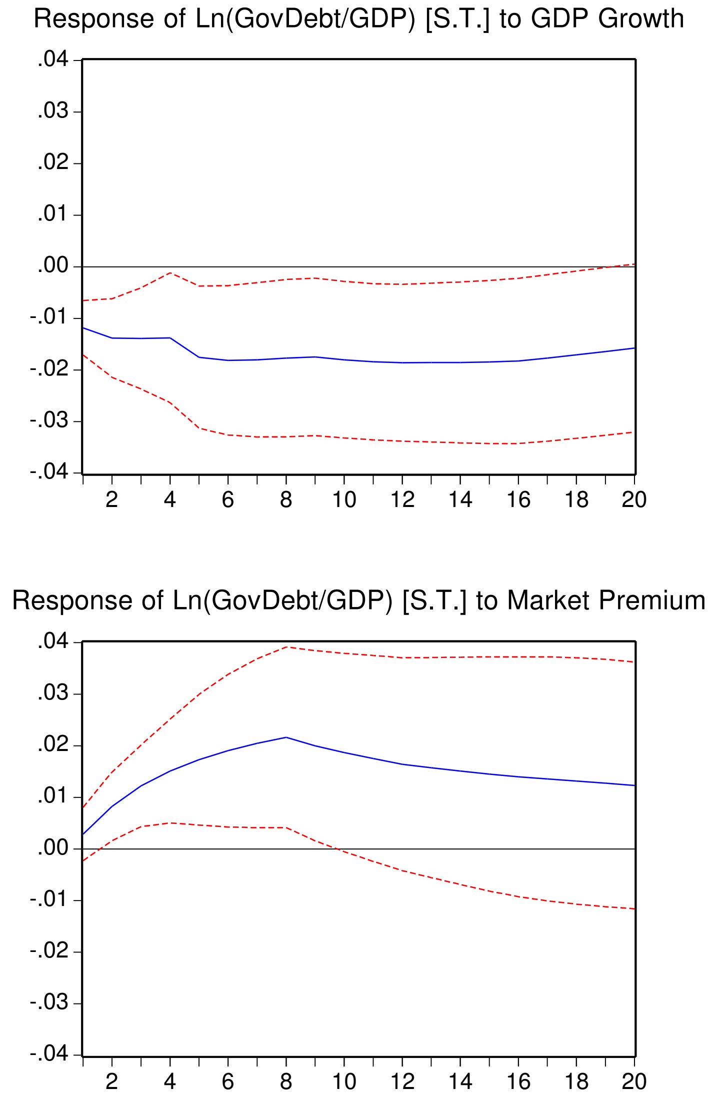

Figure 7: **Baseline VAR: Government Reaction**. We presents two impulse responses associated with the reaction of the log government debt ratio to a one standard deviation shock in either GDP growth or market uncertainty. These impulse responses are based on the baseline VAR presented in Table (7).

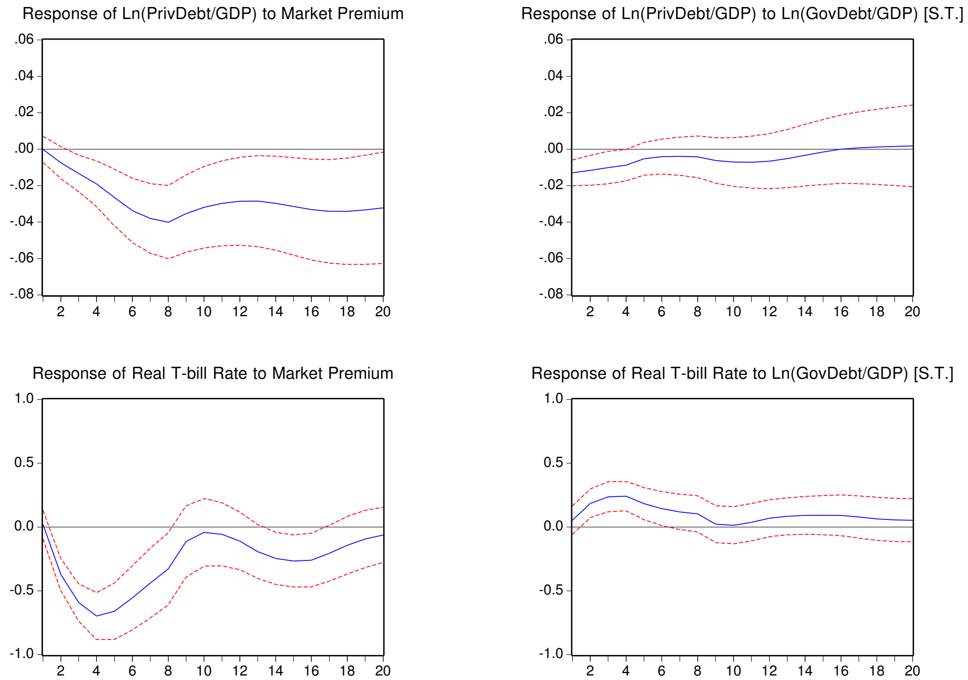

Figure 8: **Baseline VAR: Impulse Response Functions**. We provide a set of four impulse response functions of particular interest based on the estimated VAR in Table (7). To explore the predictions of our model, we focus exclusively on the responses of (1) the log of the short-term private sector debt (open-market paper) to GDP ratio and (2) the short-term real Treasury bill rate to one-standard deviation impulses in (1) the log of short-term government debt to GDP ratio and (2) the estimated market risk premium.

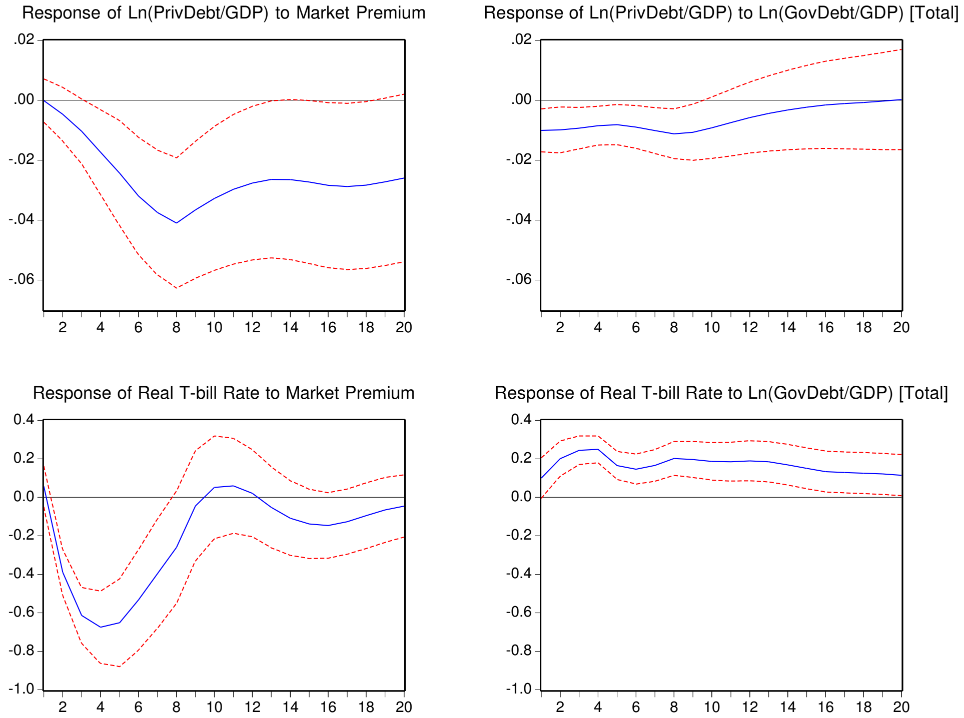

Figure 9: **Alternative VAR: Impulse Response Functions**. We provide a set of four impulse response functions of particular interest based on an alternative VAR specification (available upon request) that employs the log of the ratio of total government debt to GDP to measure public debt. To explore the predictions of our model, we focus exclusively on the responses of (1) the log of the short-term private sector debt (open-market paper) to GDP ratio and (2) the short-term real Treasury bill rate to one-standard deviation impulses in (1) the log of total government debt to GDP ratio and (2) the estimated market risk premium.

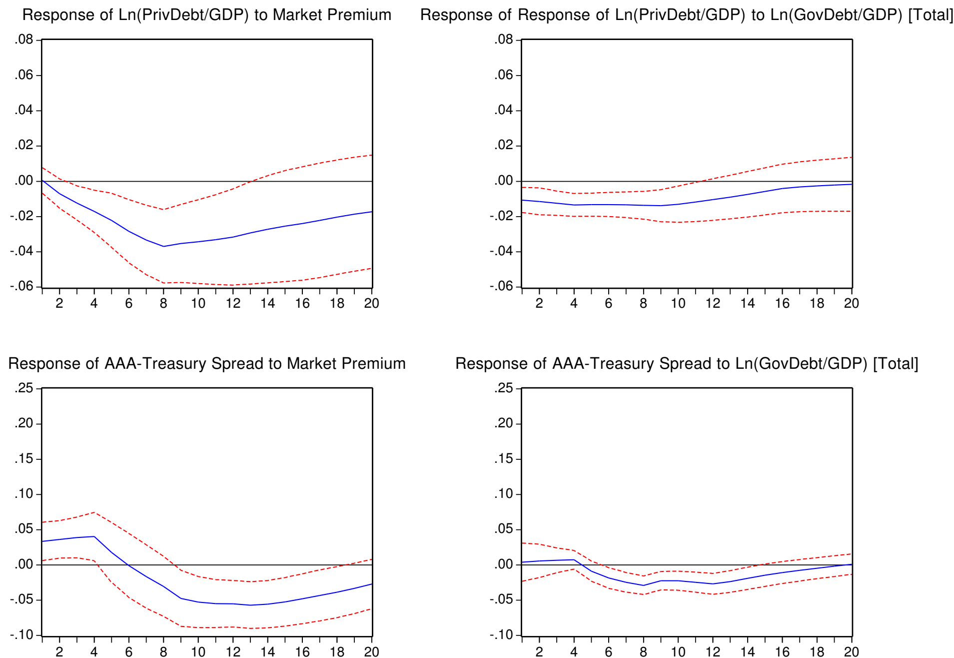

Figure 10: **Alternative VAR: Impulse Response Functions**. We provide a set of four impulse response functions of particular interest based on an alternative VAR specification (available upon request) that employs both the log of the ratio of total government debt to GDP to measure public debt and the yield spread between AAA rated bonds and Treasury bonds of similar maturity. To explore the predictions of our model, we focus exclusively on the responses of (1) the log of the short-term private sector debt (open-market paper) to GDP ratio and (2) the AAA yield spread to one-standard deviation impulses in (1) the log of total government debt to GDP ratio and (2) the estimated market risk premium.

Quantities (Ratios)

Series	Mean	Std. Deviation	ρ
GovDebt/GDP [S.T.]	0.238	0.042	0.954
GovDebt/GDP [Total]	0.388	0.110	0.967
PrivDebt/GDP	0.056	0.039	0.994
PrivDebt/GDP [Alt.]	0.186	0.066	0.993

Prices (Yields and Spreads %)

Series	Mean	Std. Deviation	ρ
Real T-bill Rate	1.329	2.144	0.873
AAA Spread	0.686	0.395	0.842

Macro-Environment %

Series	Mean	Std. Deviation	ρ
Market Premium	6.415	6.461	0.930
Real GDP growth	3.276	1.978	0.352
GDP Conditional Volatility	1.897	0.800	0.763

Table 4: **Summary Statistics: 1950-2009**. This table provides summary statistics on quarterly observations from 1950-2009 on our measures of the log of public and private debt (as a fraction of GDP), two measures of bond prices, and three measures of the macro economic environment. For public debt, we report two measures: (1) GovDebt/GDP [S.T.] represents those bills, notes, or bonds in the CRSP bond database that have less than 2-years maturity and (2) GovDebt/GDP [Total] represents the overall level of U.S. Treasury debt (of any maturity) by including all bonds covered in CRSP. For private debt, we also report two measures: (1) PrivDebt/GDP [S.T.] represents open-market paper and (2) PrivDebt/GDP [Alt.] represents the sum of quarterly observations on open-market paper, bank loans not elsewhere classified, and other loans and advances. For the price dimension, we report the real Treasury bill rates, computed as the 3-month Treasury bill rate, obtained from the Federal Reserve's release on interest rates, less a measure of expected inflation. We also report the yield spread between AAA rated bonds and Treasury bonds of similar maturity. Finally, we report summary statistics on the equity market premium (the CRSP value-weighted return less the risk-free rate), the real quarterly GDP growth rate, and conditional GDP growth volatility based on a GARCH process for the real quarterly GDP growth rate.

Coefficient	Estimate	Std. Error
α_0	-2.414	7.764
α_1	4.123	1.568
α_2	1.754	3.328
α_3	-1.184	0.727

R^2	0.043

Table 5: **Measuring Economic Uncertainty**. The expected market risk premium is the fitted process implied by the following standard return predictability regression: $Ret_{mkt,t+1} = \alpha_0 + \alpha_1 \text{MktDividendYield}_t + \alpha_2 \text{TermSpread}_t + \alpha_3 \text{TbillRate}_t + \epsilon_{t+1}$, where $Ret_{mkt,t+1}$ is the quarterly excess return on the CRSP market portfolio, $\text{MktDividendYield}_t$ is the dividend yield on the market portfolio, TermSpread_t is the term spread between long-run and short-run U.S. government bonds, and TbillRate_t is the 3-month Treasury bill rate. We employ this constructed series to directly capture risk compensation associated with economic uncertainty.

	ln(GovDebt/GDP) [S.T]	ln(GovDebt/GDP) [Total]	ln(PrivDebt/GDP)	ln(PrivDebt/GDP) [Alt.]	AAA Spread	Eq. Market Premium
ln(GovDebt/GDP) [S.T.]	1.00					
ln(GovDebt/GDP) [Total]	0.948	1.00				
ln(PrivDebt/GDP)	-0.313	-0.428	1.00			
ln(PrivDebt/GDP) [Alt.]	-0.274	-0.376	0.986	1.00		
AAA Spread	-0.298	-0.382	0.522	0.542	1.00	
Eq. Market Premium	0.520	0.563	-0.748	-0.727	-0.311	1.00

Table 6: **Correlations: 1950 - 2009** For the post-war period, we present cross-correlation correlations for the following variables: (1) GovDebt/GDP [S.T.] represents those bills, notes, or bonds in the CRSP bond database that have less than 2-years maturity; (2) GovDebt/GDP [Total] represents the overall level of U.S. Treasury debt (of any maturity) by including all bonds covered in CRSP; (3) PrivDebt/GDP [S.T.] represents open-market paper; (4) PrivDebt/GDP [Alt.], following Greenwood, Hanson, and Stein (2010a), represents the sum of quarterly observations on open-market paper, bank loans not elsewhere classified, and other loans and advances; (5) the yield spread between AAA rated bonds and Treasury bonds of similar maturity; and (6) the (estimated) market risk premium.

Table 7: **Baseline VAR** We estimate a parsimonious VAR representation of the data that incorporates the first quarter's, the first year's, and the second year's lag terms. The variables in our VAR system are (1) the real GDP growth rate, (2) the (estimated) market risk premium, (1) the log of the short-term government debt to GDP ratio, (4) the log of the private sector debt (open-market paper) to GDP ratio, and (5) the real Treasury bill rate.

	Real GDP Growth$_t$	Market Premium$_t$	Ln(GovDebt/GDP)$_t$	Ln(PrivDebt/GDP)$_t$	Real T-bill Rate$_t$
Real GDP Growth$_{t-1}$	0.1677	-19.0048	-0.3019	-1.0358	8.6109
	-0.0751	-18.2116	-0.3583	-0.4954	-7.5037
Real GDP Growth$_{t-4}$	0.0320	-32.5590	-0.4910	0.7177	0.0239
	-0.0700	-16.9740	-0.3340	-0.4617	-6.9938
Real GDP Growth$_{t-8}$	-0.0047	-22.0296	0.0331	0.6717	-7.7034
	-0.0649	-15.7273	-0.3094	-0.4278	-6.4801
Market Premium$_{t-1}$	-0.0008	0.8933	0.0026	-0.0035	-0.1906
	-0.0002	-0.0543	-0.0011	-0.0015	-0.0224
Market Premium$_{t-4}$	0.0010	-0.0856	0.0001	-0.0013	0.0393
	-0.0003	-0.0651	-0.0013	-0.0018	-0.0268
Market Premium$_{t-8}$	-0.0003	0.0718	-0.0008	0.0019	0.0598
	-0.0002	-0.0551	-0.0011	-0.0015	-0.0227
Ln(GovDebt/GDP)$_{t-1}$	-0.0057	2.8355	0.9374	-0.0191	4.2769
	-0.0107	-2.5845	-0.0509	-0.0703	-1.0649
Ln(GovDebt/GDP)$_{t-4}$	0.0213	-1.6971	-0.0263	0.0910	-1.1183
	-0.0143	-3.4570	-0.0680	-0.0940	-1.4244
Ln(GovDebt/GDP)$_{t-8}$	-0.0109	-0.5009	0.0468	-0.0421	-2.2814
	-0.0098	-2.3680	-0.0466	-0.0644	-0.9757
Ln(PrivDebt/GDP)$_{t-1}$	-0.0066	-2.0104	-0.0118	0.8571	1.4270
	-0.0066	-1.6007	-0.0315	-0.0435	-0.6595
Ln(PrivDebt/GDP)$_{t-4}$	0.0126	0.1226	-0.0136	0.0587	-0.6917
	-0.0083	-1.9988	-0.0393	-0.0544	-0.8236
Ln(PrivDebt/GDP)$_{t-8}$	-0.0072	1.3146	0.0319	0.0542	-0.8661
	-0.0055	-1.3336	-0.0262	-0.0363	-0.5495
Real T-bill Rate$_{t-1}$	0.0000	0.0713	0.0026	0.0051	0.6583
	-0.0005	-0.1118	-0.0022	-0.0030	-0.0461
Real T-bill Rate$_{t-4}$	0.0000	0.0247	-0.0006	-0.0034	-0.0398
	-0.0005	-0.1119	-0.0022	-0.0030	-0.0461
Real T-bill Rate$_{t-8}$	0.0000	-0.0440	0.0011	0.0015	0.1276
	-0.0004	-0.0887	-0.0017	-0.0024	-0.0365
Constant	0.0101	0.4389	-0.0471	-0.0240	1.6267
	-0.0077	-1.8712	-0.0368	-0.0509	-0.7710
R^2	0.249	0.898	0.945	0.997	0.847