Reconciling Bagehot
with the Fed's response to September 11 [*]

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Abstract

Bagehot (1873) states that in order to prevent bank panics a central bank should provide liquidity to the market at a "very high rate of interest." This seems to be in sharp contrast with the policy adopted by the Federal Reserve after September 11 when, for a few days, the federal funds rate was very close to zero. This paper shows that both policies can be reconciled. Bagehot had in mind a commodity money regime in which the amount of reserves available is limited. A high price for this liquidity allows banks that need it most

to self-select. In contrast, the Fed has a virtually unlimited ability to temporarily expand the money supply, so self-selection is unnecessary.

1 Introduction

This paper shows, in the context of a model economy, that two apparently incompatible policies–Bagehot's recommended policy of lending at a high interest rate and the Fed's policy of lending at a very low interest rate–can both be justified.

Bagehot (1873) states that a central bank (CB) can prevent panics by providing liquidity to the market.[1] Specifically, "there are two rules. First. That these loans should only be made at a very high rate of interest... Second. That at this rate these advances should be made on all good banking securities and as largely as the public ask for them."[2] It is widely believed that by applying this policy the Bank of England avoided panics in 1866, 1878, and 1890 (see, for example, De Kock (1974), Redish (2001)). This, in turn, explains why Bagehot's views are still influential today. As pointed out by Peter Bernstein in his foreword to a 1999 reissue of *Lombard Street*, "After nearly 150 years, [Bagehot's] wise words are still the prescription of choice for containing financial crises, as well as a handbook for avoiding them... ."

In the aftermath of the September 11 terrorist attacks in 2001, the Federal Reserve followed the second rule prescribed by Bagehot, but not the first. It lent freely and vigorously, but at very *low* interest rates. As a consequence, for a few days, from September 13 to 19, the federal funds rate approached zero on several occasions (see Markets Group of the Federal Reserve Bank of New York (2002)). The Fed's response to September 11 is generally considered to have been very successful, so one might wonder why there is such a stark difference between Bagehot's proposed policy and the Fed's action.

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[1]Throughout this paper I think of bank difficulties as arising because of a liquidity shortage and use the terms "panics" and "crises" interchangeably.

[2]Page 199.

There is theoretical support for the Fed's policy. In fact, central bank lending at an interest rate of zero in order to prevent liquidity crises seems to be robust prediction in a variety of models. For example, this policy arises in Allen and Gale (1998), Antinolfi, Huybens, and Keister (2001), Champ, Smith, and Williamson (1996), Freeman (1996), Green (1997), Rochet and Vives (2002), among others. So was Bagehot wrong?

This paper proposes a reconciliation between Bagehot's recommendation and the Fed's policy. I consider a version of the model introduced by Diamond and Dybvig (1983). Specifically, I use the model of Cooper and Ross (1998) that was recently studied by Ennis and Keister (forthcoming). In the context of that model I consider a liquidity provision policies under a commodity money regime. Because the amount of reserves is limited in a commodity money regime, there are states of nature in which some banks may not have access to these reserves. This creates strategic interactions between banks: If central bank reserves are available at a low cost, banks have an incentive to insure themselves by borrowing reserves before they know whether these reserves are needed. When banks borrow "too early," central bank reserves will not be allocated properly as some banks will have reserves they do not need while other banks are not able to acquire reserves they need. The central bank can eliminate the incentive for banks to withdraw too early by charging a high interest rate.

There is a growing literature on liquidity provision policies. See for example Cooper and Corbae (2002), Repullo (2000), Rochet and Vives (2002), Sleet and Smith (2000), Williamson (1998). These authors' work, however, does not consider the difference between commodity and fiat money regimes.

The rest of the paper is organized as follows. The next section provides some historical background. Section 3 presents the model. Section 4 consid-

ers liquidity provision in a commodity money regime while section 5 does so for a fiat money regime. Section 6 concludes.

2 Some historical background

2.1 Bagehot's recommended policy

Although many of the ideas in *Lombard Street* had been expressed before, notably by Thornton (1802), Bagehot is often credited for exposing them in a systematic way.[3] Bagehot's proposed policy contains two main elements. In times of crisis:

1) A CB should lend freely and vigorously.

2) Loans should be made at a very high interest rate.

Bagehot credits the Bank of England for having prevented a panic in 1866 by following this policy. Subsequently, in 1878 when the City Bank of Glasgow failed, and in 1890 when Baring Bank failed, the same policy is credited for preventing widespread crisis. This is in contrast to the crises of 1847 and 1857, when the Bank of England initially refused to lend, leading to bank panics.

This paper focuses on the second element of Bagehot's proposed policy: the interest rate at which loans should be made. There are, in the literature, two main arguments to justify Bagehot's claim that the CB should lend at a high interest rate. First, under the gold standard, a high rate of interest prevents a drain of gold. Second, a high rate of interest helps prevent moral hazard.

The first argument can be found in Humphrey (1975) and Humphrey and

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[3]Laidler (2002) studies the differences and the similarities between the views of Bagehot and Thornton.

Keleher (1984). They note that following Thornton (1802), Bagehot distinguishes between two types of shocks: internal (or domestic) and external (or foreign) cash drains. The former shock occurs when pessimistic depositors withdraw their deposits to hold cash and can, according to Bagehot, be countered if the CB lends vigorously. The latter shock occurs when gold flows out of England to be deposited in a foreign country. To counter such a shock the CB should raise its lending rate, so as to attract foreign gold and retain domestic gold. When the two shocks arise simultaneously, the CB should lend vigorously and at a high rate of interest.

The argument about moral hazard can be found in Sheng (1991) and Summers (1991), among others. The basic idea is that banks may take excessive risk if they know that they can borrow at a low rate during difficult times. Proponents of this view usually argue that the high interest rate Bagehot mentions is a penalty rate.

To justify his policy, Bagehot argues that "[a very high interest rate] will operate as a heavy fine on unreasonable timidity, and will prevent the greatest number of applications by persons who don't require it. The rate should be raised early in the panic, so that the fine may be paid early; that no one may borrow out of idle precaution without paying well for it; that the banking reserve may be protected as far as possible."[4]

No reference is made in this passage to an external cash drain or to moral hazard. Indeed, there are very few references to moral hazard in *Lombard Street*, and Bagehot has been criticized by Hirsch (1977) for not realizing that his proposed policy could create such a problem.[5] Instead, the quote points

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[4]Page 199.

[5]The model in this paper does not consider moral hazard problems. Martin (2006) shows that a well-designed liquidity provision policy similar to the one considered here can prevent bank panics without moral hazard.

to the need to allocate the CB liquidity in an appropriate way. Thus, my paper argues that lending at a high rate of interest allows banks to self-select.[6] Fisher (1999) seems to share this view as he notes that the high interest rate "limits the demand for credit by institutions that are not in trouble." This interpretation is also consistent with an interpretation of Goodhart (1999) that Bagehot does not propose a penalty rate.

The approach adopted by this paper is interesting for two reasons. First, from the perspective of history of thought, one wants to consider the internal consistency of Bagehot's argument. Hence, the case for a high interest rate should be made based on the type of economic mechanisms that Bagehot emphasizes, rather than on some other mechanism.[7] Second, this paper provides a formal analysis of the self-selection story which has not been studied yet.

2.2 The Fed's policy after 9-11-2001

The events of September 11 caused a breakdown in the usual means of communication between banks, and resulted in the temporary shutdown of the interbank market.[8] Some banks found themselves with high liquidity needs, while others had large excesses of liquidity. Because the interbank market

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[6]It is interesting to note that Thornton, who writes at a time during which England is off the gold standard, does not mention the need to lend at a high interest rate. This is consistent with the argument in this paper and is further support for the view that Bagehot's main concern is self-selection and not external cash drains. I am indebted to Tom Humphrey for pointing this out to me.

[7]I do not mean to suggest that moral hazard is not important in this case, only that it is not necessary to understand Bagehot. Moral hazard could provide an additional reason for a CB to charge a high rate of interest.

[8]See McAndrews and Potter (2002), Lacker (2004) for more information concerning the impact of the events of September 11 on the interbank market.

Chart 1
Total Fed Balances Around September 11, 2001

Billions of Dollars
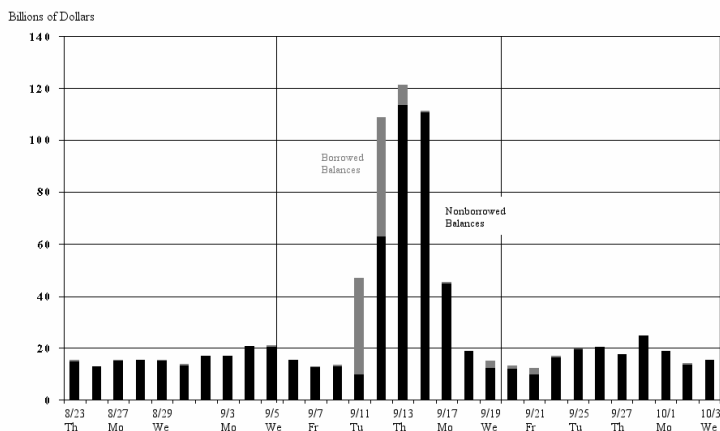
was not functioning normally, the latter banks were not able to lend to the former. To alleviate the effects of the liquidity shortage and prevent a more generalized panic, the Federal Reserve provided unusually large amounts of reserves.

The Fed typically provides liquidity to markets through the discount window (DW) and through open market operations (OMOs).[9] In an OMO the Fed provides funds to primary security dealers through a repurchase agreement (RP). The dealers lend these funds to banks on the interbank market. Ordinarily, the Fed auctions off a fixed amount of reserves and does not engage in transactions at prices that would imply a lending rate lower than its target. The DW allows banks to obtain funds directly from the Fed. At the time, the interest rate at the DW was 50 basis point lower than the federal

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[9]A third source of liquidity is float. Float is the length time between the moment a check is deposited and the moment it is available.

Chart 2
Outstanding Term and Overnight RPs Around September 11, 2001

Billions of Dollars
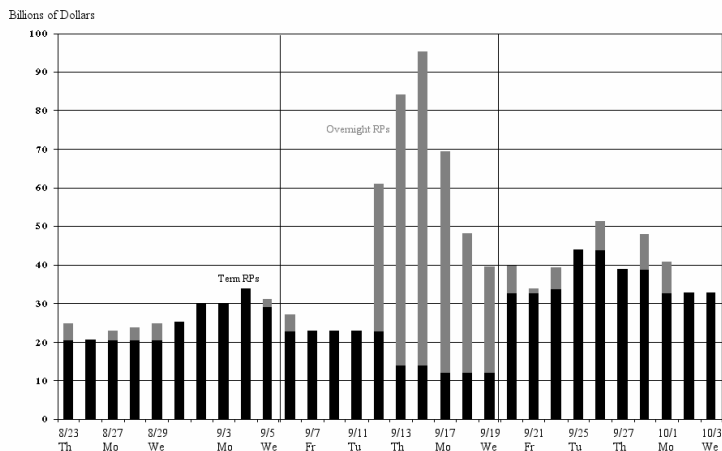
funds market target rate.[10] Banks are not allowed to lend these funds on the interbank market.

The following discussion details some of the actions of the Fed in the days following September 11. A good description of the Federal Reserve's policy after September 11 is provided by the Markets Group of the Federal Reserve Bank of New York (2002). Chart 1 shows borrowed balances (funds obtained through the DW) and nonborrowed balances (funds obtained through OMOs).[11] On September 11 and 12, large amounts of liquidity were provided through the DW because the interbank market was not function-

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[10]It was 3 percent until 9/14, 2.5 percent between 9/17 and 10/1, and 2 percent after that.

[11]Charts 1, 2, and 3 come from Markets Group of the Federal Reserve Bank of New York (2002).

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Chart 3
Federal Funds Rates Around September 11, 2001: High, Low, and Effective Rates

Percent
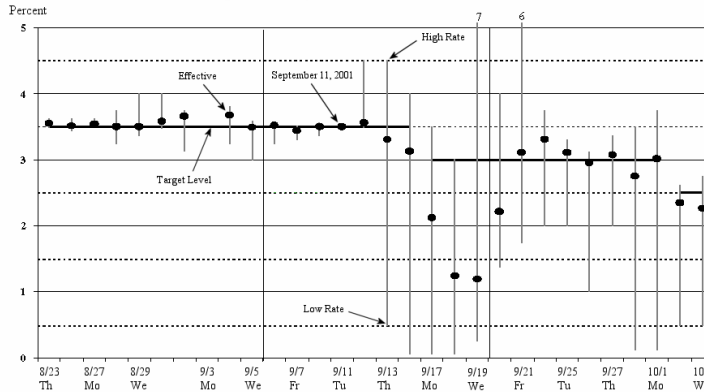
ing properly. On subsequent days, as interbank communications improved, OMOs provided much more liquidity than the DW. While the interest rate on DW loans did not change–until September 17, when the federal funds rate target was decreased by 50 basis points–banks were encouraged to borrow which made the effective cost of borrowing lower than usual. Around noon on September 11, the Board of Governors issued a press release stating: "The Federal Reserve is open and operating. The discount window is available to meet liquidity needs."

Chart 1 also shows that the Fed lent large amounts through OMOs. On September 13 and 14, the size of nonborrowed balances was more than 5 times as high as it had been in the days leading to September 11. The Fed's vigorous provision of liquidity would have satisfied Bagehot: "From Wednesday [9-12] through the following Monday [9-17], the size of open market operations were

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aimed at satisfying all the financing that dealers wished to arrange with the Desk, in order to mitigate to the extent possible the disruptions to normal trading and settlement arrangements."[12] Chart 2 shows overnight RPs and term RPs. Overnight RPs over this period can be associated with emergency lending. The size of these RPs between September 12 and 19 testify to the large amount of liquidity the Fed provided to the interbank market.

Contrary to what Bagehot would have advised, however, the Fed did not provide liquidity at a high rate: "[The Desk] had to accept the vast majority of propositions–even those offered at rates well below the new 3 percent target level–in order to arrange RPs of sufficient size."[13] The consequences of providing such large amounts of liquidity can be seen in Chart 3. The federal funds rate reached lows very close to zero on September 14, 17, and 18. The effective rate (a volume-weighted average of rates on trades arranged through the major brokers) was well below the target rate from September 17 to September 20.

The difference between Bagehot's recommended high rate of interest and the Fed's provision of liquidity at a low cost is striking and, on the face of it, puzzling. In the remainder of the paper, I argue that these differences can be explained by the fact that Bagehot had in mind a commodity money environment while the Fed operates in a fiat money environment.

3   The model

In this section, I describe a model of banks operating in a commodity money environment. In this model a central bank can prevent bank panics if it lends

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[12]Markets Group of the Federal Reserve Bank of New York (2002), page 22.

[13]Markets Group of the Federal Reserve Bank of New York (2002), page 24.

at a high interest rate.

The economy takes place at three dates, 0, 1, and 2. There is a continuum of agents called depositors and a continuum of banks. Each depositor is endowed with one unit of the economy's single consumption good at date 0 and nothing at dates 1 and 2. The depositors can be thought of as residing inside a square of sides of length 1 while the banks reside on a line of length 1. Hence, each bank has a large number of depositors.

3.1 Technologies

There are two kinds of investment technologies. The *short-term* (storage) technology yields one unit of the consumption good at date t for each unit invested at date $t - 1$, $t = 1, 2$. The *long-term* technology yields $R > 1$ units of the consumption good at date 2 for each unit invested at date 0. Liquidating the long-term technology at date 1 is assumed to carry a cost in terms of the consumption good and returns only $1 - \tau$, where $\tau \geq 0$. For example, assume that a proportion θ of the unit invested is liquidated at date 1, then the technology has return $(1 - \tau)\theta$ at date 1 and $(1 - \theta)R$ at date 2. R and τ are known by all agents.

3.2 Preferences

Households can be of two types: impatient or patient. The impatient type only derives utility from consumption at date 1, and the patient type derives utility only from consumption at date 2. Types are learned at the beginning of date 1 and are private information. Each depositor has a probability $\pi > 0$ of being impatient and a law of large number is assumed to hold so the proportion of impatient depositors in the population is also π. To keep

things as simple as possible, it is assumed that π is not a random variable. All agents know the value of π.

Let c_t denote the amount of goods consumed at date t. A depositor's expected utility is:

$$U\left(c_1, c_2, \pi\right) = \pi u\left(c_1\right) + \left(1 - \pi\right) u\left(c_2\right).$$

Patient agents can use the storage technology to store goods they obtain at date 1. Alternatively, it could be assumed that they derive utility from the sum of their subperiod 1 and subperiod 2 consumption. The function u is strictly increasing, strictly concave, and satisfies $u(0) = 0$.

3.3 Spatial and informational constraints

As in Wallace (1988), depositors are assumed to be spatially separated and unable to meet or trade with each other at date 1. At date 0, agents can deposit their endowment in any bank. Depositors can withdraw their funds either at date 1 or at date 2. Those depositors who withdraw at date 1 arrive at the bank in random order. A sequential service constraint is imposed so that the bank must pay depositors as they arrive, without knowing how many depositors will ultimately show up. Following Freeman (1988), I assume that the number of withdrawals at date 1 is not observable to depositors immediately. Depositors can only observe whether or not their bank runs out of funds. Hence, bank payments at date 1 cannot depend on the number of depositors who withdraw at that date or on the order in which depositors arrive at the bank. Depositors would not accept such pattern of bank payments because they would be unable to verify whether they receive the correct payment. Hence banks cannot offer contracts involving suspension of convertibility.

3.4 The deposit contract

Banks offer a contract that promises a fixed payment of c_1 goods at date 1. All depositors withdrawing at date 1 receive this amount unless the bank runs out of funds. In such a case, depositors arriving at the bank after it has run out of funds receive nothing. All depositors who withdraw at date 2 receive an equal share of the resources left in the bank. The payment promised to such depositors is denoted by c_2.

As is standard in this kind of model, if everyone believes that only impatient depositors will withdraw at date 1, then it is individually rational for patient depositors to withdraw at date 2. However, if everyone believes that all patient depositors will withdraw at date 1, then it is individually rational for them to do so, provided that $c_1 > 1 - i\tau$. Indeed, in that case liquidating the long-term technology does not provide enough resources for the bank to give c_1 to all depositors at date 1. The resulting allocation is associated with a bank run. This paper focuses on economies in which the above inequality holds.

There exists a third equilibrium to the post-deposit game. If the right fraction of patient depositors withdraws at date 1, then all depositors receive c_1 whether they withdraw at date 1 or at date 2. This equilibrium can be associated with a partial bank run since some, but not all, patient depositors withdraw early. I do not consider partial bank runs in this paper, as does most of the literature.

If the probability of a bank run is perceived to be strictly positive, the bank will take that into account when choosing its investment in the long-term and the storage technology.

3.5   Sunspot

As in Ennis and Keister (forthcoming) I assume that bank runs are triggered by a sunspot.[14] Depositors have the following beliefs: If a sunspot is observed, everyone believes patient depositors withdraw at date 1, provided it is individually rational for them to do so. Otherwise everyone believes they withdraw at date 2.[15]

Let $q > 0$ denote the expected probability that an individual bank will be affected by a sunspot. I allow for the possibility that the depositors of only a fraction of banks observe the sunspot when it occurs.[16] One possible case is that with probability q a sunspot occurs and all banks are affected. It could also be the case that every period a sunspot is observed by a fraction q of all banks. Linear combinations of these two cases are possible as well. From the perspective of an individual bank, only the expected probability of a sunspot matters.

3.6   The banks' problem

Banks are assumed to maximize profits. Because of perfect competition, banks offer, in equilibrium, a deposit contract that maximizes the expected utility of depositors.[17] Depositors' beliefs are coordinated by a sunspot as described above and depositors choose when to withdraw so as to maximize

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[14]This is a common approach adopted, among others, by Benthal et al. (1990), Cooper and Ross (1998) and (2002), Freeman (1988), and Peck and Shell (2003). Also, Ennis (2003) argues that empirical evidence is not inconsistent with the idea that bank run can be triggered by sunspots.

[15]Most of the literature assigns a probability zero to partial bank runs.

[16]For example, because banks are in different regions.

[17]Allen and Gale (1998), Cooper and Ross (1998), Schreft and Smith (1998), among others, adopt this approach.

their utility. Hence, impatient depositors always withdraw at date 1 since they get no utility from consuming later. Patient depositors will withdraw at date 1 if there is a sunspot and at date 2 otherwise. In other words, all consumption enjoyed by depositors who withdraw at date 2 comes from investment in a long-term technology.[18] The bank's problem can be written

$$\max(1-q)[\pi u(c_1) + (1-\pi)u(c_2)] + q\hat{\pi}u(c_1)$$

subject to

$$\pi c_1 \leq 1 - i, \tag{1}$$

$$(1-\pi)c_2 \leq Ri, \tag{2}$$

$$\hat{\pi} = \min\{\frac{1-i\tau}{c_1}, 1\}, \tag{3}$$

$$c_1, c_2, i \geq 0, \tag{4}$$

where i denotes the investment in the long-term technology. Hence, in case of a bank run, a depositor receives c_1 with probability $\hat{\pi}$ and nothing otherwise.

The bank can choose to offer a deposit contract such that bank runs never occur. As shown by Cooper and Ross (1998), if q is sufficiently small banks offer a deposit contract that allows runs. I focus on such cases in this paper. Ennis and Keister (forthcoming) show that there exists a unique solution to the bank's problem.

4   Liquidity provision with commodity money

This section considers a liquidity provision policy by a central bank (CB). The CB is assumed to be an agent that has the ability to tax the endowment

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[18]I assume, throughout the paper, than banks never invest in excess liquidity, Ennis and Keister (forthcoming) prove that this is indeed the case whenever $u(c) = c^\alpha/\alpha$, where $\alpha < 1$.

of depositors and can invest in the storage technology. The objective of the CB is to maximize the expected utility of depositors.

The CB can prevent sunspot-driven bank runs if it is able to provide enough goods to banks affected by a sunspot at date 1. Indeed, bank runs occur because depositors are concerned that their bank may have to liquidate its long-term investment in order to pay depositors at date 1. If the bank can guarantee that this will not be the case, then bank runs are avoided. When the CB charges a low interest rate, banks have an incentive to borrow before they know if they need reserves. This can lead to a misallocation of those scarce reserves. If the CB charges a high enough interest rate, this incentive disappears.

A question that naturally arises is: Why would there be a need for a CB to supply liquidity since banks can already choose run-proof contracts? The reason is that while banks face idiosyncratic risk the system may or may not face aggregate risk. Consider the two extreme cases: If all banks in the system are affected by a sunspot, then the liquidity need of an individual bank and of the system are the same. A CB can play no role in this case. However, if in every period a given fraction of banks are affected by a sunspot, then the system faces no aggregate risk and a CB may be able to help.[19]

4.1   Central bank lending

The CB is assumed to be unable to invest in the long-term technology. This reflects the fact that the CB is not able to identify good projects as well

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[19]It might be possible for a market to play the role that the CB plays in this paper (see, for example, Allen and Gale 2004). I do not consider this possibility as CB provided liquidity appears to be the historically relevant case. A number of arguments have been made to explain why a CB may be better at providing liquidity than an interbank market (see, for example, Goodhart 1988, or Rochet and Vives 2004).

as banks can.[20] Further, for each unit of good taxed, the CB obtains $\delta < 1$ reserves, so that it is costly for the CB to raise funds. I argue, in the appendix, that with these assumptions there is no loss of generality in restricting the CB to make loans to banks at some net interest rate r.

Assume that with probability ε all banks observe a sunspot. With probability $1-\varepsilon$, only a fraction q' of banks observe a sunspot. Let $q \equiv (1-\varepsilon)q'+\varepsilon$. I maintain the assumption that q is sufficiently small so that individual banks do not offer a run-preventing contract. Also, I assume that it is too costly for the CB to prevent a run at all banks (see appendix). Since raising funds is costly, the CB will acquire enough reserves to prevent bank runs when only a fraction q' of banks are affected. When, with probability ε, all banks are affected, the CB is unable to prevent bank runs at some banks.[21]

In this section, I consider the case where $\varepsilon = 0$, so that the economy faces no aggregate uncertainty. This is the best case scenario for CB since it knows exactly the liquidity needs of the banking system. I show that even in that case, the CB may need to charge a high interest rate to provide the right incentives for banks. The results extend in a straightforward way to the case where $\varepsilon > 0$ by continuity.

The timing of events, in every period, is as follows: First, at date zero, the CB levies a lump-sum tax T from the endowment of all consumers. Next,

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[20]Assuming the CB is able to invest in a long-term technology with return $\tilde{R} < R$ would not modify my results.

[21]This appears to be the relevant case for the Bank of England historically. Also note that while the Bank of England had, in principle, the ability to expend its issue of notes and suspend their convertibility, the assumption that the CB does not hold enough reserves to prevent certain panics remains valid if the cost of suspension of convertibility (real or perceived) is high enough. There is some evidence that this cost was indeed high; for example the panics of 1847 and 1857 subsided only after the Chancellor of the Exchequer announced it would cover the cost of the Bank of England if its Issue Department expanded its note issue without gold backing and was sued.

consumers deposit their net-of-tax endowment in banks. At the beginning of date 1, before the sunspot occurs, banks can choose to borrow from the CB. Then, banks' depositors may observe a sunspot and depositors learn whether they are impatient or not. Again, banks are able to borrow from the CB, this time knowing whether or not their depositors have observed a sunspot. Then withdrawals take place. At date 2, CB loans are repaid. Next, the CB distributes all its assets equally to banks that have not been affected by a run. Finally the assets of each bank are divided equally among all agents who did not withdraw at date 1. The CB does not observe the sunspot and does not know exactly when the sunspot occurs. Hence, the CB is unable to determine whether a bank that asks for a loan has been affected by a sunspot or not.[22]

I assume that CB loans are senior to deposits, so that banks must make sure that they have enough resources to repay their debt to the CB before patient depositors can withdraw.[23] Let θ denote the fraction of the long term-technology that is liquidated at date 1, L denote the size of the CB loan, and r denote the (net) interest rate on the loan. Then the loan must satisfy

$$(1 - \pi)c_1 \leq (1 - \tau)\theta i + L, \tag{5}$$

$$(1 + r)L \leq Ri(1 - \theta), \tag{6}$$

and $L \geq 0$. The first equation indicates that the sum of the CB loan and the goods obtained from the partial liquidation of the long-term technology are enough to provide c_1 to all patient depositors withdrawing at date 1. The

[22]Alternatively, it could be assumed that there is an early and a late sunspot and that the CB is unable to determine whether or not a bank has observed the early sunspot.

[23]Note that if CB liquidity provision is feasible and desirable under this assumption, then it is also feasible and desirable if deposits are senior to CB loans.

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second equation assures that the bank has enough goods left to repay the CB loan. Note that a bank would never borrow funds from the CB if the gross interest rate $1 + r$ is greater than $R/(1 - \tau)$. Hence, $1 + r < R/(1 - \tau)$ is assumed throughout.

Since raising funds is costly for the CB, it only taxes the minimum necessary. For the remainder of the paper L will denote the smallest loan necessary to prevent bank runs. The CB holds qL reserves and makes a loan of L to each bank that wants to borrow until there are no reserves left. Each bank is assumed to have the same probability of arriving at the CB early enough to borrow.

To find L I solve equations 5 and 6 at equality, and use $\pi c_1 = 1 - i - T$, to get

$$L = \max \left\{ 0, \frac{R \left[(1 - i - T)\frac{1-\pi}{\pi} - (1 - \tau)i \right]}{R - (1 + r)(1 - \tau)} \right\}. \tag{7}$$

To prevent panics at a mass q of banks, the CB must raise taxes $\delta T = qL$. Combining these two equations, I get

$$L = \max \left\{ 0, \frac{R \left[(1 - i)\frac{1-\pi}{\pi} - (1 - \tau)i \right]}{R - (1 + r)(1 - \tau) + R\frac{1-\pi}{\pi}\frac{q}{\delta}} \right\}. \tag{8}$$

L increases in r since a large r means, everything else constant, that more resources must be used to repay the debt.

The largest loan a bank might need is $L_{max} \equiv (1 - \pi)c_1$. In this case, the bank has resources $Ri = (1 - \pi)c_2$ to repay the loan. It follows that if the CB charges an interest rate $1 + r \leq c_2/c_1$ then the bank can repay any loan of size less than or equal to L_{max}.

Now I can write the expected utility of depositors at a bank under different assumptions about banks' behavior. A bank can choose to try to borrow from the CB before it knows whether its depositors observe a sunspot or it can choose to wait. The bank chooses the action that maximizes the expected

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utility of its depositors, given its belief about what other banks do. Since banks are ex-ante identical, they will either all try to borrow early, or all choose to wait, unless they are indifferent between the two. If banks are indifferent, I assume that they choose the same action.

I denote by E the case where all banks choose to borrow early from the CB and by W the case where all banks wait. If all banks borrow early, the expected utility of depositors in a bank that is able to obtain reserves from the CB is denoted $EU_{B/E}$, while the expected utility of depositors in a bank that is unable to obtain reserves is denoted $EU_{NB/E}$. Similarly, in the case where banks wait, the expected utility of depositors in a bank that borrows from the CB is $EU_{B/W}$ while the expected utility of depositors in a bank that does not borrow is $EU_{NB/W}$.

Consider the case where banks only borrow if their depositor observes a sunspot. With probability q, the bank's depositors observe the sunspot and the bank borrows from the CB. Since banks that do not observe the sunspot do not borrow, the bank obtains enough reserves to prevent a panic with probability 1. Depositors' expected utility is given by

$$EU_{B/W}(i) = \pi u \left(\frac{1 - i - \frac{1}{\delta}qL}{\pi} \right) + (1 - \pi)u \left(\frac{Ri}{1 - \pi} + \frac{-rL + (1 + r)qL}{1 - \pi} \right). \tag{9}$$

The consumption of patient depositors corresponds to the goods invested by the bank in the long-term technology, minus the interest rate paid on the CB loan, plus a share of the goods the CB redistributes to all banks.

With probability $1 - q$, the bank's depositors do not observe the sunspot and the bank does not borrow from the CB. In this case, the depositor's expected utility is given by

$$EU_{NB/W}(i) = \pi u \left(\frac{1 - i - \frac{1}{\delta}qL}{\pi} \right) + (1 - \pi)u \left(\frac{Ri}{1 - \pi} + \frac{(1 + r)qL}{1 - \pi} \right). \tag{10}$$

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For any i, $EU_{NB/W}(i) \geq EU_{B/W}(i)$ for all $r \geq 0$, with a strict inequality if and only if $r > 0$. This immediately implies the following result.

Proposition 1 *It is a Nash equilibrium for all banks to wait until they know if their depositors observe a sunspot before borrowing from the CB.*

Indeed, if all banks wait, and if $r > 0$, then it is a dominant strategy for a bank to wait and borrow only if its depositors observe a sunspot. If $r = 0$, then banks are indifferent between waiting or borrowing early, since there is no cost of borrowing.

Now consider the case where all banks choose to borrow early. I assume that all banks have the same probability, q, of being able to obtain reserves from the CB. If a bank is able to borrow from the CB, it will not be affected by a bank run, whether its depositors observe a sunspot or not. The depositor's expected utility in this case is

$$EU_{B/E}(i) = \pi u \left(\frac{1 - i - \frac{1}{\delta}qL}{\pi} \right) + (1-\pi)u \left(\frac{Ri}{1 - \pi} + \frac{-rL + (1+r)\frac{q}{1-q(1-q)}L}{1 - \pi} \right).$$
(11)

Note that $EU_{B/E}(i) > EU_{B/W}(i)$ because if all banks borrow early, some banks are unable to obtain liquidity from the central bank even though they need it and thus fail. The fraction of such banks is equal to $q(1 - q)$. The share of the CB's reserves that would be distributed to these banks at the end of date 2 is shared equally among all surviving banks.

With probability $1 - q$, the bank is unable to obtain reserves from the CB. The bank's depositors observe a sunspot with probability q, so their expected utility is given by

$$
\begin{aligned}
EU_{NB/E}(i) &= (1 - q)\left\{ \pi u \left(\frac{1 - i - \frac{1}{\delta}qL}{\pi} \right) + (1 - \pi)u \left(\frac{Ri}{1 - \pi} + \frac{(1+r)\frac{q}{1-q(1-q)}L}{1 - \pi} \right) \right\} \\
&\quad + q\hat{\pi}u \left(\frac{1 - i - \frac{1}{\delta}qL}{\pi} \right).
\end{aligned}
$$
(12)

Since $\hat{\pi}u[(1-i-\frac{1}{\delta}qL)/\pi] < EU_{NB/W}(i)$, and since $EU_{NB/W}(i) < EU_{B/W}(i)$ for $r = 0$, it immediately follows that $EU_{NB/E}(i) < EU_{B/E}(i)$ if r is small enough. This can be summarized in the following proposition.

Proposition 2 *It is a Nash equilibrium for all banks to try to borrow early from the CB, if the CB charges a small enough interest rate.*

Hence, if r is low enough, there are multiple equilibria. If all banks are expected to try to borrow early, then it is an equilibrium to do so, while if all banks expected to wait, then it is a equilibrium to wait. Now, I want to show that it is possible for the CB to eliminate the equilibrium of proposition 2 by charging a high enough interest rate. Note that it is a dominant strategy for banks to wait if $EU_{NB/E}(i^e) > EU_{B/E}(i^e)$, where i^e denotes equilibrium investment. Recall that

$$EU_{B/E}(i) = \pi u\left(\frac{1-i-\frac{1}{\delta}qL}{\pi}\right) + (1-\pi)u\left(\frac{Ri}{1-\pi} + \frac{-rL+(1+r)qL}{1-\pi}\right)$$
(13)

and

$$\begin{aligned} EU_{NB/E}(i) &= (1-q)\left[\pi u\left(\frac{1-i-\frac{1}{\delta}qL}{\pi}\right) + (1-\pi)u\left(\frac{Ri}{1-\pi} + \frac{(1+r)qL}{1-\pi}\right)\right] \\ &+ q\hat{\pi}u\left(\frac{1-i-\frac{1}{\delta}qL}{\pi}\right). \end{aligned}$$
(14)

Inspection of $EU_{NB/E}(i)$ and $EU_{B/E}(i)$ reveals that for any $r > 0$, $EU_{NB/E}(i) > EU_{B/E}(i)$ if $q = 0$. By continuity, the inequality will hold for small values of q. This can be summarized in the following proposition.

Proposition 3 *If the CB charges a high enough interest rate, it is a a dominant strategy for all banks to wait before they borrow from the CB, provided q is not too big.*

21

Note that the CB does not need to hit a particular rate of interest "just right" in order to provide incentive for banks to wait. Any interest rate, above some level, will do. Of course, the CB would like to set the lowest rate that provides the proper incentives since depositors' expected utility decreases with r.

A natural question to ask is how small must q be? The answer turns out to depend on the third derivative of the utility function. Note that for any given i, it is possible to find r large enough so that $EU_{NB/E}(i) > EU_{B/E}(i)$, regardless of q.[24] However, the equilibrium value of i chosen by banks changes with r. If i does not change very much with changes in r, then q does not need to be very small. It can be shown that i does not change very much with changes in r if $u'''(\frac{Ri}{1-\pi} + \frac{(1+r)qL}{1-\pi})$ is small.

4.2 Welfare

It remains to be shown that the CB can improve welfare by making loans despite the fact that it must charge a high interest rate and that the consumption of some patient depositors comes from stored goods. I provide an example in this section.

Following Ennis and Keister (forthcoming), I assume that the utility function is given by $u(c) = c^\alpha/\alpha$. Parameters are set at the following values: $\alpha = 0.5$, $\pi = 0.8$, $R = 1.2$, $q = 0.05$, $\delta = 0.95$, and $\tau \to 1$.[25] Rather than jointly solve for the optimal i and the optimal r, I choose $r = 0.15$ and solve for i. I show that $EU_{NB/E} > EU_{B/E}$ in equilibrium. Further, for these parameters and the chosen interest rate, welfare is higher with central bank

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[24]However, q cannot be too large since the CB cannot charge an interest rate greater than c_2/c_1.

[25]Note that $\tau \to 1$ implies that banks cannot offer run preventing contacts.

lending than without. Finally, I show that welfare is higher with central bank lending than if the CB provides goods to all banks.

Under the above assumptions, it can be verified that $L \rightarrow \frac{19}{75}(1-i)$, so that

$$
\begin{aligned}
EU_E(i) &= qEU_{B/E}(i) + (1-q)EU_{NB/E}(i) \\
&= \pi u\left(\frac{\frac{74}{75}(1-i)}{\pi}\right) \\
&\quad + (1-\pi)\left[qu\left(\frac{1.2i - \frac{9.25}{100}L}{1-\pi}\right) + (1-q)^2\left(\frac{1.2i + \frac{5.75}{100}L}{1-\pi}\right)\right].
\end{aligned}
$$

The expected utility EU_E is maximized at $i \approx 0.2062$. Since $\tau \rightarrow 1$, then

$$
EU_{NB/E}(i) > EU_{B/E}(i) \Leftrightarrow (1-q)u(\frac{Ri + (1+r)qL}{1-\pi}) > u(\frac{Ri + (1+r)qL - rL}{1-\pi}).
$$
(15)

This inequality holds at the maximizing i.

It remains to be verified that CB lending at rate $r = 0.15$ is preferred to the absence of CB intervention or to a policy in which the CB would hold enough reserves to prevent panics at all banks, and charge an interest rate of zero. It can be verified that

$$
EU_W(i = 0.2062) = qEU_{B/W}(i = 0.2062) + (1-q)EU_{NB/W}(i = 0.2062) \approx 2.037.
$$
(16)

Let EU_{NCB} denote the expected utility of depositors when the CB does not intervene.

$$
EU_{NCB}(i) = (1-q)\left[\pi u\left(\frac{1-i}{\pi}\right) + (1-\pi)u\left(\frac{Ri}{1-\pi}\right)\right] + q\pi u\left(\frac{1-i}{\pi}\right).
$$
(17)

In this case, the expected utility of depositors is maximized for $i \approx 0.2131$ and $EU_{NCB}(i = 0.2131) \approx 2.0165 < EU_W(i = 0.2062)$. Hence, CB lending at $r = 0.15$ yields more expected utility that no CB lending.

As is shown in the appendix, the CB can prevent bank runs at all banks if it raises taxes $T \geq 1 - i$. Let EU_{NR} denote the expected utility of depositors when the CB raises enough funds to prevent runs at all banks.[26]

$$EU_{NR}(i) = \pi u \left(\frac{1 - i - T(1 - \delta)}{\pi} \right) + (1 - \pi)u \left(\frac{Ri}{1 - \pi} \right). \qquad (18)$$

In this case, the expected utility of depositors is maximized for $i = 0.24$ and $EU_{NR}(i = 0.24) = 2 < EU_W(i = 0.2062)$. Hence, expected utility is higher if the CB holds enough reserves to prevent panics at only q banks, and charges a high interest rate for loans, than if the CB holds enough reserves to prevent panics at all banks and charges no interest rate.

5 Liquidity provision with fiat money

Martin (2006) studies liquidity provision policy with fiat money in the same type of model as the one presented in this paper and shows that the optimal policy is to lend at zero interest rate. The intuition is that, with fiat money, it is always possible for the CB to have enough reserves for all banks.[27]

One could argue that the model of bank runs in this paper is not appropriate to study the events following September 11, 2001. The reason might be that the optimal policy in case of a liquidity shortage depends on the specific cause of this shortage. Indeed, in 2001 the liquidity shortage did not arise as a consequence of pessimistic expectations but because of the physical destruction of some communication infrastructures.

Champ, Smith, and Williamson (1996) present a model in which liquidity needs arise because of random "relocation" shocks. One interpretation of

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[26]Note that EU_{NR} is

[27]An additional benefit of fiat money is that the CB does not need to hold physical resources. With commodity money some patient agents consume stored goods rather then goods from the long-term technology.

such shocks could be the kind of communication breakdown that followed the terrorist attacks. In that model also, the optimal response to a liquidity shock is to provide money freely at an interest rate of zero. Williamson (2004) provides yet another example of an environment in which liquidity needs do not arise because of pessimistic expectations. In that model also, CB lending should take place at zero interest rate. While this does not constitute a proof, it does suggest that a policy resembling the one suggested by Bagehot is unlikely to be optimal in a fiat money environment.

6   Conclusion

This paper proposes a reconciliation between Bagehot's recommended policy of lending reserves at a high interest rate and the Federal Reserve response to September 11, 2001, which consisted of providing reserves at a very low cost.[28] The key is to recognize that Bagehot had in mind a commodity money world, while the Fed operates in a fiat money world.

In a commodity money regime, lending funds at a zero interest rate cannot prevent bank panics, while Bagehot's policy can. The reason for this result is that when reserves are scarce strategic interactions arise between banks that potentially need CB reserves. If liquidity is available at a low cost, banks have an incentive to insure themselves by acquiring liquidity before they know whether they will need it. A consequence is that liquidity may not be allocated efficiently. If the CB charges a high enough rate, then banks

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[28]The model implies that the Fed could have done even better if it had lowered the interest rate on discount window loans. In 2002 the Fed modified the way in which the discount window operates. The interest rate is now set to be 100 basis points above the federal funds rate target. However, it is specified that the interest rare can be lowered in special circumstances.

always wait until they know whether they need liquidity before attempting to borrow. I show that CB lending at an interest rate high enough to provide the right incentives to banks can yield higher expected welfare to depositors than no CB lending.

7 Appendix

If the CB can invest in the long-term technology, and there were no cost of raising funds, then bank runs could be prevented in the following way: The CB invests all the resources it taxes in the long-term technology. Banks invest all the deposits they receive in the storage technology and the deposit contract promises $c_2 = 0$. All the assets in the CB are given to depositors who do not withdraw at date 1. There would be no bank runs in this case since all patient depositors know that the long-term projects held by the CB will not be liquidated.

If the CB cannot invest in the long-term technology, but raising funds is not costly, then bank runs can be prevented in a different way: The CB invests all the resources it taxes in the storage technology. Banks invest all the deposits they receive in the long-term technology and the deposit contract promises $c_1 = 0$. All assets in the CB are given to depositors who "withdraw" at date 1. Again, bank runs are prevented in this case because banks never need to liquidate the long-term technology.

Under the scheme proposed above, the bank does not need to set c_1 equal to zero. It is enough to set c_1 low enough that $c_1 \leq 1 - \tau i - T$. Recall that

the bank's deposit contract is

$$c_1 = \frac{1 - i - T}{\pi}, \tag{19}$$

$$c_2 = \frac{Ri}{1 - \pi}. \tag{20}$$

It follows that $c_1 \leq 1 - \tau i - T$ if and only if

$$i \geq \frac{(1 - \pi)(1 - T)}{1 - \tau\pi}. \tag{21}$$

The expected utility of depositors under the CB scheme is given by

$$EU_{NR} = \pi u\left(\frac{1 - i - T(1 - \delta)}{\pi}\right) + (1 - \pi)u\left(\frac{Ri}{1 - \pi}\right), \tag{22}$$

where T is implicitly defined by equation 21 at equality. The expected utility of depositors if the CB does no intervene is given by

$$EU_{NCB} = (1 - q)\left[\pi u\left(\frac{1 - i}{\pi}\right) + (1 - \pi)u\left(\frac{Ri}{1 - \pi}\right)\right] + q\hat{\pi}u\left(\frac{1 - i}{\pi}\right). \tag{23}$$

Inspection of these two expressions reveals that for any $\delta < 1$, there exists a q small enough that $EU > EU^{CB}$. This paper focuses on the case where it is too costly for the CB to provide goods to all banks.

A transfer scheme between the CB and banks can be described as follows. At date 1, the CB transfers goods to banks that ask for it, until it runs out. At date 2, the transfers between the CB and banks is conditional on whether or not a bank received a transfer from the CB at date 1.[29] Note

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[29]In principle, the transfer at date 2 could also depend on whether a bank tried to obtain reserves from the CB but was not able to. This case can be ruled out using the following argument: If such a transfer was positive, all banks would have an incentive to borrow to receive the transfer. This cannot be optimal for the CB. If the transfer is negative, one can assume that banks observe that the CB runs out of reserves as soon as it does, and do not request reserves in this case. Hence, the CB is unable to distinguish between banks that intended to borrow but were not able to and banks that did not intend to borrow.

that the transfers can be positive or negative. It can be shown that any set of transfers can be rewritten as a combination of two transfers: First, a transfer from banks that obtained reserves from the CB at date 1 to the CB and, second, a transfer from the CB to all banks, regardless of whether they obtained reserves at date 1. Since all banks borrow the same amount from the CB, the transfers at date 1 and 2 between the CB and banks that obtain funds can be though of as a loan. The ratio of the date 2 transfer to the date 1 transfer, in absolute value, is the gross interest rate on the loan.

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