Structural Dynamic Analysis of Systematic Risk

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Paris, March 2016

In this paper we . . .

- propose a **structural dynamic factor model (SDFM)** for stock returns featuring:
  - a multiasset framework;
  - a latent common factor;
  - stochastic volatilities:
    - for the common factor;
    - for each financial institution;
  - nonlinear distance-to-default effects on stock values;
- provide an **indirect inference** method to estimate our model;
- validate our estimation via Monte-Carlo simulations;
- use stock returns of 10 financial institutions between 2000 and 2015;
- **rank the different banks according to default risk and speculation risk**.

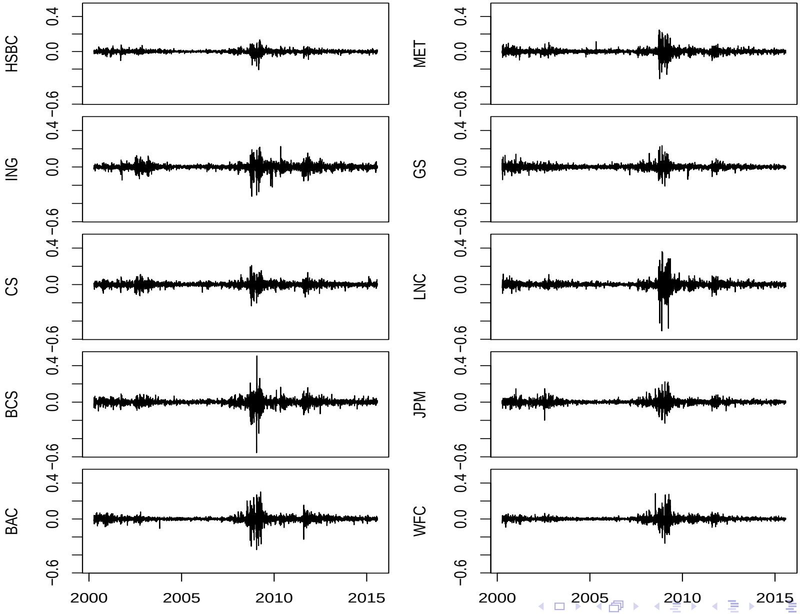
Outline

1. The data
2. A simple dynamic factor model without option of default
3. The structural dynamic factor model with default risk
4. Estimation results

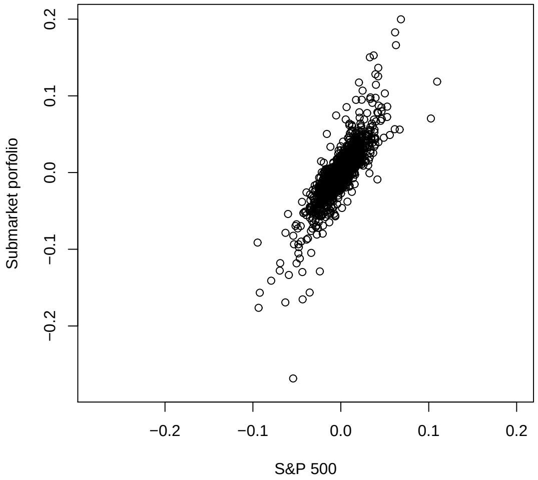
1. The data

10 daily logreturns between April 6 2000 and July 31 2015:

1. MetLife Inc. (MET)
2. ING Group (ING)
3. Goldman Sachs Group Inc. (GS)
4. Lincoln National Corp. (LNC)
5. HSBC Holdings plc. (HSBC)
6. JPMorgan Chase & Co. (JPM)
7. Bank of America Corp. (BAC)
8. Credit Suisse Group AG (CS)
9. Wells Fargo & Co. (WFC)
10. Barclays plc. (BCS)


HSBC
ING
CS
BCS
BAC
MET
GS
LNC
JPM
WFC
0.4
0.0
−0.6
2000
2005
2010
2015

Submarket not representative of the entire market



$\rightarrow$ slope $= 1.5$

Regression on the submarket portfolio logreturns

	α	β
MET	0.00025	0.97036
ING	-0.00020	1.15132
GS	0.00011	0.83225
LNC	0.00004	1.24654
HSBC	0.00002	0.59355
JPM	0.00002	0.98732
BAC	-0.00017	1.16722
CS	-0.00017	0.93051
WFC	0.00023	0.93459
BCS	-0.00013	1.18635

$\rightarrow$ Static linear factor model

However. . .

The static linear model does not capture volatility features.

2. A simple dynamic model without option of default

Jump to SDFM

Y_{it}, $i = 1, \ldots, n$, $t = 1, \ldots, T$ are the returns at date t for stock i.

$$\begin{cases} Y_{it} = \alpha_i + \beta_i F_t + \sigma_{it} \epsilon_{it} \,, & i = 1, \ldots, n \,, \\ F_t = \gamma F_{t-1} + \eta_t u_t \,, \end{cases}$$

- F_t is a common unobservable **linear factor**;
- $\sigma^2_{it} \sim \text{ARG}(\tilde{\delta}, \tilde{\rho}, \tilde{c})$ additional **nonlinear factors** where ARG stands for **Autoregressive Gamma**:
  - $\sigma^2_{it} \sim \text{Gamma}(\tilde{\delta} + z_t, \tilde{c})$ and
  - $z_t \sim \text{Poisson}(\tilde{\rho}\sigma^2_{i,t-1}/\tilde{c})$;
- $\eta^2_t \sim \text{ARG}[\delta, \rho, (1-\rho)(1-\gamma^2)/\delta]$;
- ϵ_{it} and u_t are independent $\mathcal{N}(0, 1)$.

Estimation

Auxiliary method of moments estimator

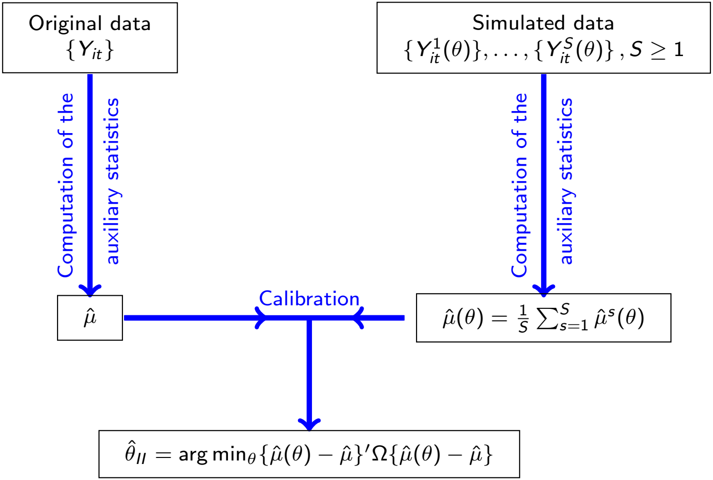
A method of moments estimator exists:

$$\hat{\mu} = (\hat{\alpha}_1, \ldots, \hat{\alpha}_n, \hat{\beta}_1, \ldots, \hat{\beta}_n, \hat{\gamma}, \hat{\delta}, \hat{\rho}, \hat{\hat{\delta}}, \hat{\hat{\rho}}, \hat{\hat{c}})',$$

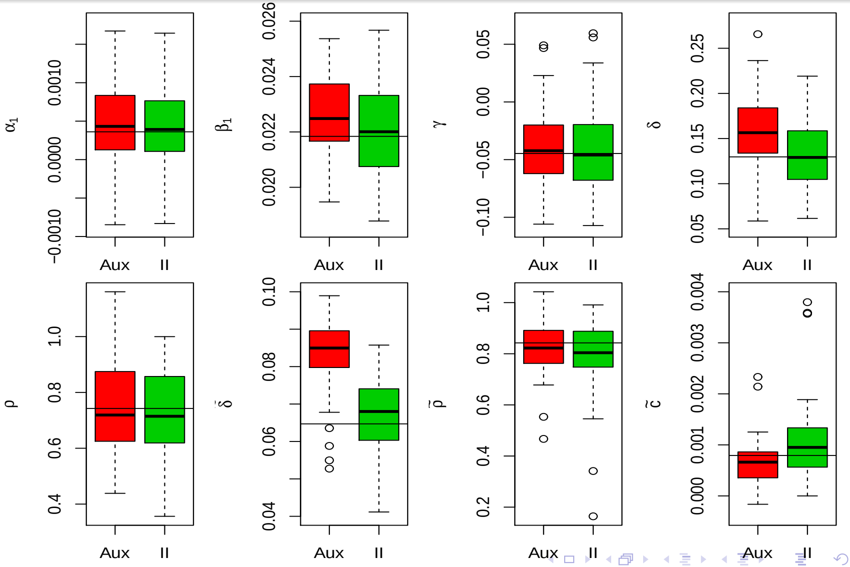
but is inconsistent for a finite number of firms n.

Idea: adjust for bias by **indirect inference**.

Indirect inference (Gouriéroux et al., 1993; Smith, 1993)



Monte Carlo Results, $S = 10$



Empirical results

	Aux	II
α_{MET}	0.000395	0.000362
α_{ING}	-0.000032	0.000059
α_{GS}	0.000232	0.000200
α_{LNC}	0.000227	0.000118
α_{HSBC}	0.000111	0.000163
α_{JPM}	0.000164	0.000074
α_{BAC}	0.000005	-0.000047
α_{CS}	-0.000028	0.000047
α_{WFC}	0.000366	0.000372
α_{BCS}	0.000043	-0.000053

	Aux	II
β_{MET}	0.021806*	0.021840*
β_{ING}	0.025872*	0.026084*
β_{GS}	0.018702*	0.018239*
β_{LNC}	0.028012*	0.028496*
β_{HSBC}	0.013338*	0.012607*
β_{JPM}	0.022187*	0.022368*
β_{BAC}	0.026229*	0.026669*
β_{CS}	0.020910*	0.020599*
β_{WFC}	0.021002*	0.020969*
β_{BCS}	0.026659*	0.026857*
γ	-0.053297	-0.044679
δ	0.162371*	0.129703*
ρ	0.787698*	0.742651*
$\tilde{\delta}$	0.077740*	0.064702*
$\tilde{\rho}$	0.815843*	0.842556*
$\tilde{c}$	0.000687	0.000792

Goodness-of-fit of the linear factor

	Static model	Dynamic model
MET	0.62513	0.25282
ING	0.64071	0.25170
GS	0.58649	0.22973
LNC	0.62914	0.25735
HSBC	0.58584	0.22771
JPM	0.71357	0.26583
BAC	0.70451	0.26740
CS	0.60075	0.24063
WFC	0.69037	0.25688
BCS	0.62410	0.24433

↑ Regression R^2

↖ $$R_i^2 = \frac{\sum_{t=1}^{T} (\beta_i F_t)^2}{\sum_{t=1}^{T} \{(\beta_i F_t)^2 + (\sigma_{it} \epsilon_t)^2\}}$$

However, in order to rank financial institutions, we need to introduce **default risk**. . .

3. The structural dynamic factor model (SDFM)

- Stock as a call option → pricing formula under no arbitrage opportunity (Black and Scholes, 1973; Merton, 1974):

$$P_{it} = \overset{Q}{\mathbb{E}} \left[\exp(-r_t)(A_{i,t+1} - L_{i,t+1})^+ \mid \mathcal{I}_t \right]$$

  - P_{it} equity value and r_t is the riskfree rate
  - A_{it} is the asset value and L_{it} the level of liability
  - $\mathcal{I}_t$ information available at date t and Q a pricing measure

- the asset value is a GBM with **intraday instantaneous volatility** ω_{it}

$$P_{it} = g^{BS}(A_t, L_{i,t|t+1}, r_t, \omega_{it}^2) = A_t \Phi(d_{1,t}) - L_{i,t|t+1} \Phi(d_{2,t}) \exp(-r_t) \,,$$

  - $d_{1,t} = \frac{1}{\omega_{it}} \left\{ \log\left(\frac{A_{it}}{L_{i,t|t+1}} \right) + r_t + \frac{\omega_{it}^2}{2} \right\}$;
  - $d_{2,t} = d_{1,t} - \omega_{it}^2$.

Simplified Merton's formula with $r_t = 0$ and $\omega_{it} = \omega \; \forall t$

$$\begin{cases} Y_{it} = h\left[\log\left(A_{it}/L - 1\right); \omega\right] - h\left[\log\left(A_{i,t-1}/L - 1\right); \omega\right] \;, \\ a_{it} = \alpha_i + \beta_i F_t + \sigma_{it}\epsilon_{it} \;,\; i = 1, \ldots, n \,, \\ F_t = \gamma F_{t-1} + \eta_t u_t \;, \end{cases}$$

- A_{it} is the asset value of firm i at date t;
- L liabilities assumed constant through i and t;
- $a_{it} = \Delta \log(A_{it}/L - 1)$;
- $h\left[\log\left(A_{it}/L - 1\right); \omega\right] =$ $\log\left\{\frac{A_{it}}{L}\Phi\left[\frac{1}{\omega}\log\left(\frac{A_{it}}{L}\right) + \frac{\omega}{2}\right] - \Phi\left[\frac{1}{\omega}\log\left(\frac{A_{it}}{L}\right) - \frac{\omega}{2}\right]\right\}$.

Remarks:

1. if ω is small

$$Y_{it} \approx a_{it} \,,$$

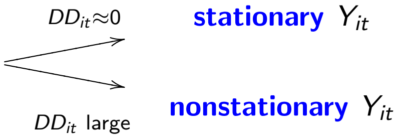
   a_{it} is the log excess asset liability ratio growth which is **stationary**
   $\rightarrow$ in the extreme case we are back to our simple model

   Back to simple dynamic model

2. in general

$$Y_{it} = a_{it} + DD_{it}$$

   where DD_{it} is the **risk premia of the insurance against default**.



Three levels of information

NATURE:
knows
$\{\sigma_{it}, \eta_t, F_t, a_{it}, \omega_{it}\}$

↓

INVESTORS:
$\{A_{it}, L, \omega_{it}, Y_{it}\}$

↓

ECONOMETRICIAN:
observes asset returns Y_{it} and wants to infer $\{A_{it}/L, DD_{it}\}$ for ranking purposes

Problem

The model is highly nonlinear in the parameters + state variables.

Challenging:

- estimation (no direct estimation)
  $\rightarrow$ we use **indirect inference**;
- filtering to estimate the distribution of the hidden states $x_t = (F_t, \eta_t, \{\sigma_{it}\}, \{a_{it}\})$:

$$f(x_t | Y_t)\,,\ \forall t$$

  $\rightarrow$ $f(Y_t|x_t)$ unavailable in closed form and we use **accurate ABC filtering** (Calvet and Czellar, 2015);
- smoothing to estimate

$$f(x_t | Y_T)\,,\ \forall t$$

  $\rightarrow$ $f(x_t|x_{t-1})$ is not available in closed-form and we develop **ABC smoothing**.

Estimation via indirect inference

New moment needed

An additional moment needed to identify ω.

Naive estimator of intraday volatility:

$$\tilde{\omega} = \frac{1}{n} \sum_{i=1}^{n} \sqrt{\frac{1}{T-1} \sum_{t=1}^{T} \left(\exp(Y_{it}) - \overline{\exp(Y_i)} \right)^2}.$$

$\tilde{\omega}$ is the interperiod volatility averaged on the banks.

Estimation results

	No option of default	SDFM
α_{MET}	0.000362	0.000606
α_{ING}	0.000059	0.000198
α_{GS}	0.000200	0.000596
α_{LNC}	0.000118	0.000358
α_{HSBC}	0.000163	-0.000122
α_{JPM}	0.000074	-0.000212
α_{BAC}	-0.000047	0.000440
α_{CS}	0.000047	0.000159
α_{WFC}	0.000372	0.000694
α_{BCS}	-0.000053	-0.000950

	No option of default	SDFM
β_{MET}	0.021840	0.026920
β_{ING}	0.026084	0.034525
β_{GS}	0.018239	0.020364
β_{LNC}	0.028496	0.035498
β_{HSBC}	0.012607	0.016979
β_{JPM}	0.022368	0.036311
β_{BAC}	0.026669	0.034107
β_{CS}	0.020599	0.026767
β_{WFC}	0.020969	0.023378
β_{BCS}	0.026857	0.050174
γ	-0.044679	-0.054902
δ	0.129703	0.161184
ρ	0.742651	0.869339
$\tilde{\delta}$	0.064702	0.077236
$\tilde{\rho}$	0.842556	0.856746
$\tilde{c}$	0.000792	0.000786
ω		1.861301

Default risk measures

- the excess asset liability ratio

$$EAL_{it} = \widehat{A_{it}/L} - 1\,,$$

- the risk premia of the insurance against default

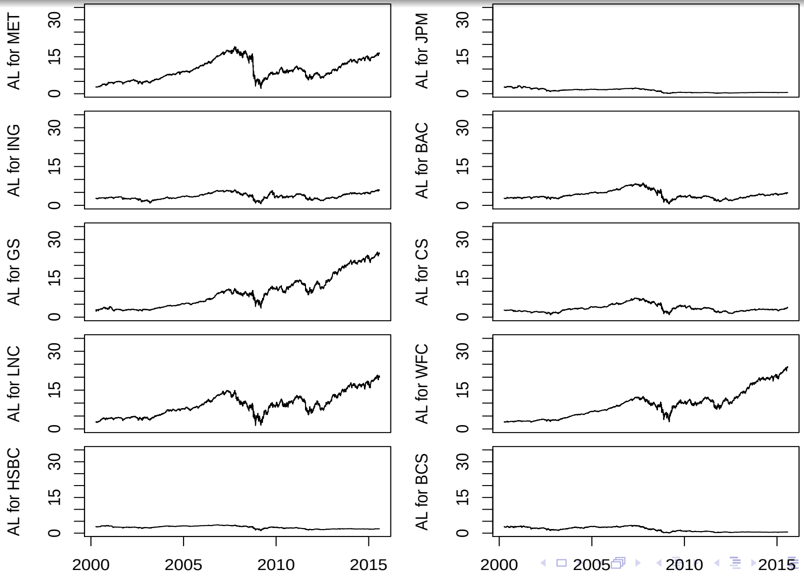
$$DD_{it} = \hat{h}_i[\log(\widehat{A_{it}/L} - 1); \hat{\omega}_{II}] - \log(\widehat{A_{it}/L} - 1)\,.$$

- the change in the risk premia is defined by:

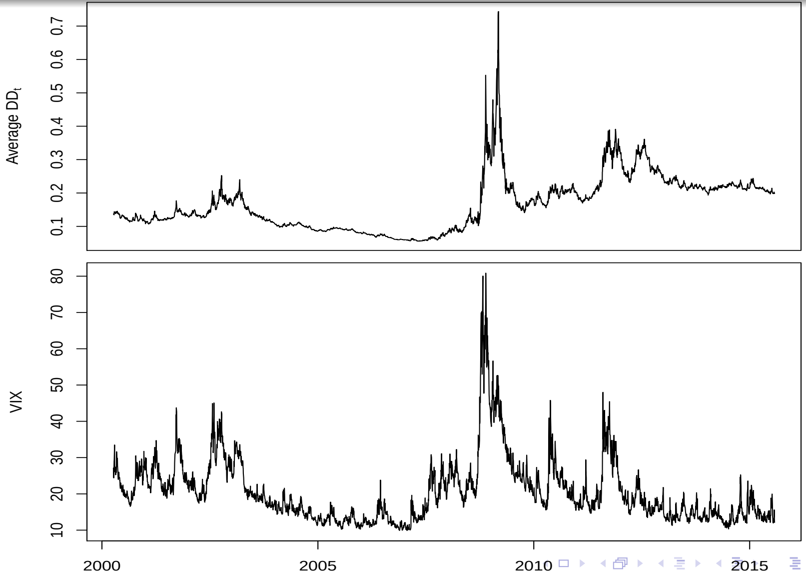
$$\begin{aligned}\Delta DD_{it} = \hat{h}_i[\log(\widehat{A_{it}/L} - 1); \hat{\omega}_{II}] - \hat{h}_i[\log(\widehat{A_{i,t-1}/L} - 1); \hat{\omega}_{II}] \\ - \Delta\log(\widehat{A_{it}/L} - 1)\,.\end{aligned}$$

They are implied measures

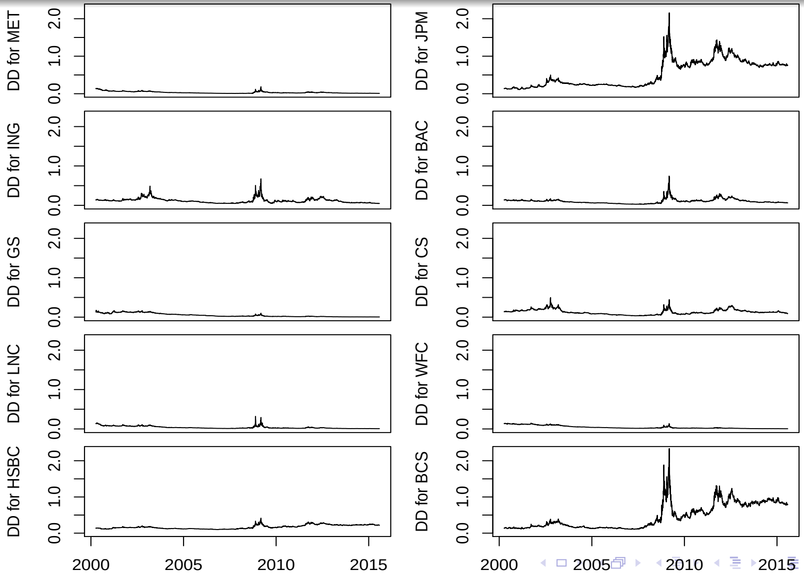
Asset/liability ratios



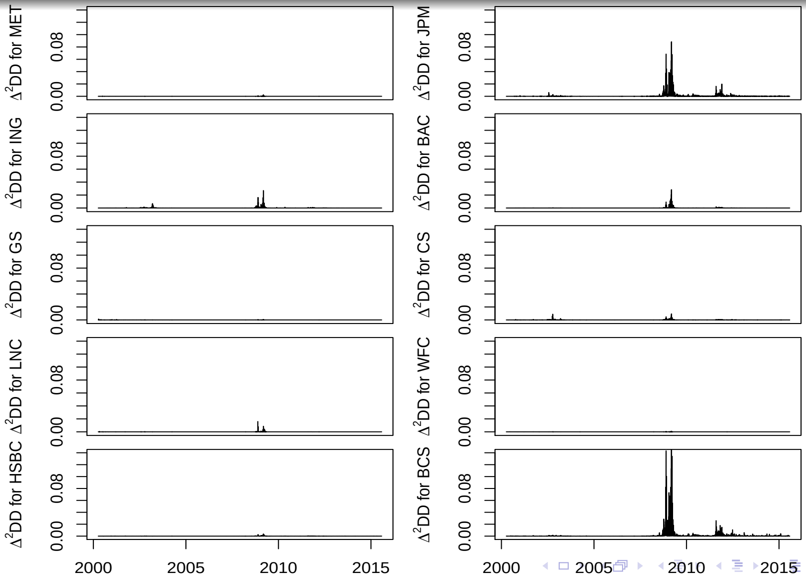
Average risk premia of the insurance



Risk premia of the insurance



Squared change in default risk premia



Rankings before the crisis (Aug 2006-July 2007)

	Solvency	Cost of Default Ins.	Speculative Assets
MET	10 (15.979390)	10 (0.009880)	10 ($1.75\,10^{-8}$)
ING	4 (5.489695)	4 (0.053089)	3 ($6.73\,10^{-7}$)
GS	7 (9.481001)	7 (0.023619)	6 ($2.05\,10^{-7}$)
LNC	9 (13.427560)	9 (0.013245)	8 ($4.26\,10^{-8}$)
HSBC	3 (3.325922)	3 (0.105957)	4 ($6.10\,10^{-7}$)
JPM	1 (2.099304)	1 (0.188462)	1 ($5.99\,10^{-6}$)
BAC	6 (7.660523)	6 (0.032434)	7 ($1.80\,10^{-7}$)
CS	5 (6.439518)	5 (0.042771)	5 ($3.31\,10^{-7}$)
WFC	8 (11.074720)	8 (0.018223)	9 ($3.84\,10^{-8}$)
BCS	2 (3.05096)	2 (0.118780)	2 ($4.71\,10^{-6}$)

Rankings during the crisis (Aug 2008-July 2009)

	Solvency	Cost of Default Ins.	Speculative Assets
MET	9 (7.09690)	8 (0.050289)	8 ($6.36\,10^{-5}$)
ING	4 (2.364981)	4 (0.202826)	4 ($8.85\,10^{-4}$)
GS	10 (7.366679)	10 (0.038123)	10 ($1.89\,10^{-5}$)
LNC	7 (5.471638)	7 (0.069957)	5 ($3.26\,10^{-4}$)
HSBC	3 (1.951240)	3 (0.215971)	7 ($1.65\,10^{-4}$)
JPM	1 (0.459097)	1 (0.971626)	2 ($5.07\,10^{-3}$)
BAC	5 (2.618628)	5 (0.197779)	3 ($9.19\,10^{-4}$)
CS	6 (2.995614)	6 (0.149538)	6 ($3.09\,10^{-4}$)
WFC	8 (7.024829)	9 (0.042289)	9 ($3.07\,10^{-5}$)
BCS	2 (0.605754)	2 (0.843310)	1 ($9.35\,10^{-3}$)

Rankings after the crisis (Aug 2010-July 2011)

	Solvency	Cost of Default Ins.	Speculative Assets
MET	7 (9.759512)	7 (0.022262)	8 ($3.14\,10^{-7}$)
ING	6 (3.804111)	6 (0.090215)	3 ($9.39\,10^{-6}$)
GS	10 (12.907010)	10 (0.014171)	10 ($9.71\,10^{-8}$)
LNC	9 (11.169130)	9 (0.018100)	7 ($4.41\,10^{-7}$)
HSBC	3 (2.119382)	3 (0.186953)	5 ($5.40\,10^{-6}$)
JPM	1 (0.449330)	1 (0.832362)	2 ($1.55\,10^{-4}$)
BAC	4 (3.177631)	4 (0.113556)	4 ($8.58\,10^{-6}$)
CS	5 (3.335567)	5 (0.106028)	6 ($4.97\,10^{-6}$)
WFC	8 (10.872050)	8 (0.018783)	9 ($1.85\,10^{-7}$)
BCS	2 (0.688051)	2 (0.593960)	1 ($2.08\,10^{-4}$)

Rankings last month (July 2015)

	Solvency	Cost of Default Ins.	Speculative Assets
MET	7 (16.238560)	7 (0.009504)	7 ($3.20\,10^{-8}$)
ING	6 (5.873010)	6 (0.048071)	6 ($1.41\,10^{-6}$)
GS	10 (24.491080)	10 (0.004592)	10 ($5.24\,10^{-9}$)
LNC	8 (20.098160)	8 (0.006551)	8 ($2.87\,10^{-8}$)
HSBC	3 (1.814079)	3 (0.223024)	4 ($4.12\,10^{-6}$)
JPM	2 (0.501605)	2 (0.762814)	2 ($7.37\,10^{-5}$)
BAC	5 (4.763128)	5 (0.065000)	5 ($1.81\,10^{-6}$)
CS	4 (3.575994)	4 (0.096361)	3 ($6.75\,10^{-6}$)
WFC	9 (23.531360)	9 (0.004940)	9 ($7.17\,10^{-9}$)
BCS	1 (0.472679)	1 (0.798621)	1 ($1.28\,10^{-4}$)

Correlation of rankings

	Solvency	Cost of def. ins.	Speculation
Solvency	1		
Cost of def. ins.	0.99596	1	
Speculation	0.88687	0.89091	1

The ranking can also be done over firms and time, by defining ratings from the historical distribution of returns along percentile segments.

Ratings after the crisis (Aug 2010-July 2011)

	Solvency	Cost of Default Ins.	Speculative Assets
MET	bb	bb	cc
ING	cc	cc	c^-
GS	bbb	bbb	ccc
LNC	bb	bb	cc
HSBC	c^-	c^-	c^-
JPM	c^-	c^-	c^-
BAC	c	c	c^-
CS	c	c	c^-
WFC	bb	bb	cc
BCS	c^-	c^-	c^-

Thank you!

Additional slides

ABC filter (Calvet and Czellar 2011, 2014; Jasra et al. 2011, 2012)

Step 1 (Sampling): Simulate a **state-observation pair** $(\tilde{x}_t^{(n)}, \tilde{Y}_t^{(n)})$ from $f_{X,Y}(\cdot|x_{t-1}^{(n)}, Y_{1:t-1})$, $n = 1, \ldots, N$.

Step 2 (Importance weights): Observe Y_t and compute

$$\omega_t^{(n)} = \frac{1}{h_t^{n_Y}} K\left(\frac{\tilde{Y}_t^{(n)} - Y_t}{h_t}\right), \ n = 1, \ldots, N.$$

Step 3 (Multinomial resampling): Draw $x_t^{(1)}, \ldots, x_t^{(N)}$ from $\tilde{x}_t^{(1)}, \ldots, \tilde{x}_t^{(N)}$ with probabilities $\frac{\omega_t^{(1)}}{\sum_i \omega_t^{(i)}}, \ldots, \frac{\omega_t^{(N)}}{\sum_i \omega_t^{(i)}}$.

Quasi-Cauchy kernel $K(u) = (1 + c\, u^\top u)^{-(n_Y+3)/2}$ and plug-in bandwidth (Calvet and Czellar, 2014)

n_Y	c	Plug-in bandwidth $h_t^*(N)$
1	$\pi^2/4$	$\left(\frac{5\pi^4}{128\, N\, \tilde{P}_t}\right)^{1/5}$
2	$2\pi/3$	$\left(\frac{\pi^2}{3N\tilde{P}_t}\right)^{1/6}$
3	$(\pi^2/4)^{2/3}$	$\left(\frac{21\pi^{8/3}}{32\cdot 4^{4/3} N\tilde{P}_t}\right)^{1/7}$
4	$2\pi/\sqrt{15}$	$\left(\frac{2\pi^2}{15N\tilde{P}_t}\right)^{1/8}$

$$\tilde{P}_t = \frac{2\,\mathrm{tr}(\tilde{\Sigma}_t^{-2}) + [\mathrm{tr}(\tilde{\Sigma}_t^{-1})]^2}{2^{n_Y+2}\,\pi^{n_Y/2}\,[\det(\tilde{\Sigma}_t)]^{1/2}}$$

ABC Smoothing Algorithm (current paper)

Step 0 (Particle filtering): Construct an ABC filter $\{x_t^{(i)}\}_{t=1,\dots,T}^{i=1,\dots,N}$.

Step 1 (Positioning of the backward simulation): Choose $\tilde{x}_T = x_T^{(i)}$ with probability $1/N$.

Step 2 (Backward simulation): For each t and i

(i) generate a pseudo-particle $\hat{x}_{t+1}^{(i)}$ from $f(x_{t+1}|x_t^{(i)})$;

(ii) compute the importance weights:

$$\omega_{t|t+1}^{(i)} = \frac{K_{h_t}\left(\tilde{x}_{t+1} - \hat{x}_{t+1}^{(i)}\right)}{\sum_{i'=1}^{N} K_{h_t}\left(\tilde{x}_{t+1} - \hat{x}_{t+1}^{(i')}\right)}, \quad i \in 1, \dots, N;$$

(iii) choose $\tilde{x}_t = x_t^{(i)}$ with probability $\omega_{t|t+1}^{(i)}$.

Step 3 (Path drawing): $\tilde{x}_{1:T} = (\tilde{x}_1, \dots, \tilde{x}_T)$ is an approximate realization from $f(x_{1:T}|Y_{1:T})$.

$$\hat{\alpha} = \overline{Y}\,,\ \hat{\beta}_i = \frac{1}{T}\sum_{t=1}^{T} Z_{it}\hat{F}_t\,,\ \hat{\gamma} = \frac{1}{T-1}\sum_{t=2}^{T}\hat{F}_t\hat{F}_{t-1}\,.$$

$$\hat{\delta} = \left\{(T-1)^{-1}\sum_{t=2}^{T}\widehat{\eta_t u_t}^4/[3(1-\hat{\gamma}^2)^2]-1\right\}^{-1}\,,$$

$$\hat{\rho} = \hat{\delta}\left((T-2)^{-1}\sum_{t=3}^{T}\widehat{\eta_t u_t}^2\widehat{\eta_{t-1}u_{t-1}}^2/[(1-\hat{\gamma}^2)^2]-1\right)\,.$$

$$\hat{\hat{\delta}} = \left\{\frac{T^{-1}\sum_{t=1}^{T}\sum_{i=1}^{n}\widehat{\sigma_{it}\epsilon_{it}}^4}{3n\hat{A}^2}-1\right\}^{-1}\,,$$

$$\hat{\hat{\rho}} = \hat{\hat{\delta}}\left[\frac{(T-1)^{-1}\sum_{t=2}^{T}\sum_{i=1}^{n}\widehat{\sigma_{it}\epsilon_{it}}^2\widehat{\sigma_{i,t-1}\epsilon_{i,t-1}}^2}{n\hat{A}^2}-1\right]\,,$$

$$\hat{\hat{c}} = (1-\hat{\hat{\rho}})\hat{A}/\hat{\hat{\delta}}\,.$$